Question 1 (10 marks)
Provide a polynomial algorithm for the necessary winner problem for the plurality rule.

Question 2 (10 marks)
Show that for elections with three alternatives, iterative voting restricted to best responses need not converge for the Borda rule. Assume that the initial profile is the truthful profile and that ties are broken lexicographically, just as we did in class.

Question 3 (10 marks)
A weak Condorcet winner is an alternative that wins or draws against any other alternative in pairwise majority contests. Just like a (normal) Condorcet winner, a weak Condorcet winner need not exist for all preference profiles. Unlike a Condorcet winner, however, when it does exist, a weak Condorcet winner need not be unique.

In the context of voting in combinatorial domains, show that when voters model their preferences using the language of prioritised goals and each voter only specifies only a single goal, then there must always be a weak Condorcet winner.