Computational Social Choice 2021

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Plan for Today

The first part of the course was about the classical model of voting and some of the seminal results in social choice theory.

The next part of the course is about very recent (and still tentative) work on studying innovative ideas for democratic decision making.

We begin by introducing liquid democracy, which has been proposed as a compromise between direct and representative democracy.

The central idea is that voters should be allowed to delegate their right to vote to a proxy (another voter) and that delegations are transitive.
History

The idea of using flexible forms of vote delegation can be traced back all the way to the writings of Charles L. Dodgson in the 1880s.

In the 1960s, anticipating widely available computer technology, Gordon Tullock and James C. Miller made similar proposals.

Since the early 2000s such proposals have become more frequent and more specific. Around the same time, these ideas were first put into practice, notably by several “Pirate” and “Internet” parties.


Aside: The Ostrogorski Paradox

Voters in a district need to choose between representatives L and R. On each area of policy you prefer one of these two representatives. You vote for the one you agree with on a majority of areas:

<table>
<thead>
<tr>
<th></th>
<th>Fiscal</th>
<th>Immigration</th>
<th>Transport</th>
<th>Vote</th>
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</thead>
<tbody>
<tr>
<td>Working Class (30%)</td>
<td>L</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Middle Class (40%)</td>
<td>L</td>
<td>L</td>
<td>R</td>
<td>L</td>
</tr>
<tr>
<td>The Nobility (30%)</td>
<td>R</td>
<td>L</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
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 Majority  

Thus: A majority of voters disagree on a majority of policy areas with the representative elected—by the majority rule.

Nice illustration of certain shortcomings of representative democracy.

The Model

The voters in $N = \{1, \ldots, n\}$ need to choose an alternative from $A$. Many (most?) papers to date focus on scenarios with $|A| = 2$.

Each voter $i \in N$ either (i) reports a preference $R_i$ (e.g., from $\mathcal{L}(A)$) or (ii) delegates her right to vote to another voter $j \in N \setminus \{i\}$.

$\Rightarrow$ delegation graph $\langle N, \rightarrow \rangle$, with the sinks being the casting voters

If $\langle N, \rightarrow \rangle$ is acyclic, we can construct a profile $R = (R_1, \ldots, R_n)$, by setting $R_i := R_{i^*}$ for voter $i$ and the casting voter $i^*$ with $i \rightarrow^* i^*$.

We can then apply our favourite voting rule to this preference profile.

When there are cycles (which is usually considered highly undesirable), the simplest solution is to assume that the voters involved abstain.

Remark: Richer models, where voters can distribute their right to vote over several proxies or where voters can rank several proxies, have also been considered (in part to address the challenge of cycle formation).
Research Questions

What is a reasonable model for how delegation graphs are formed?

- How do voters choose? Link between preferences and delegation?
- Delegation on everything / specific issues / policy areas?

What can be said about the structure of delegation graphs?

- How should we deal with cycles? Interpret them as abstentions?
- Should we be concerned about extreme concentrations of power?
- Should we impose restrictions on delegation for better control?

What voting rule should we use to aggregate cast preferences?

- Should the structure of the graph matter (or just \# leaves)?
- Normative characterisation? Epistemic characterisation?

How does liquid democracy perform relative to other approaches?

- Compared to direct / representative democracy?
- Compared to proxy voting without transitivity?
Examples for Research Contributions

Let us consider three examples for research contributions:

- some strategic considerations during delegation graph formation
- difficulties in interpreting preferences induced by delegation
- epistemic accuracy of liquid democracy systems
The Paradox of Unwelcome Delegation

Paradoxically, a casting voter may have an incentive to reject another voter who would want to delegate her voting rights to her.

This example illustrates this phenomenon for the plurality rule:

\[
\begin{align*}
  a &\succ b &\succ c \\
  b &\succ a &\succ c \\
  c &\succ b &\succ a
\end{align*}
\]

\[\uparrow\quad \quad \quad \quad \uparrow\quad \quad \quad \quad \uparrow\]

2 supporters 2 supporters 3 supporters

Here the first casting voter will be better off if her two supporters delegate their voting rights to the second casting voter instead!

Nabakoff (1988) credits Thomas Schelling for having observed this phenomenon (in case a cup rule is used to aggregate preferences).

Pervasiveness of the Paradox

This problem of voters not always wanting to receive endorsements is closely related to the well-known problem of strategic manipulation:

Take any case of a successful manipulation in the standard model where the manipulator $i$’s true preference $R_i$ is shared by voter $j$.

Now imagine $i$ is considering to delegate her right to vote to $j$. Then $j$ would rather not receive that endorsement!

Recall that—by Gibbard-Satterthwaite and Duggan-Schwartz—all reasonable voting rules are subject to manipulation (for $|A| \geq 3$).

Kotsialou and Riley (2020) demonstrate a similar phenomenon even for $|A| = 2$ for a richer model allowing for ranked delegations.

Pairwise Delegation

A common idea in proposals for liquid democracy is to allow voters to delegate to different peers for issues regarding different policy areas. Similarly, when $|A| > 2$, we may want to allow voters to delegate each pairwise ranking decision to a different peer. Challenges:

- delegation cycles $\Rightarrow$ induced preferences may be incomplete
- delegation to multiple voters $\Rightarrow$ cycles in induced preferences

Brill and Talmon (2018) propose several approaches:

- Allow for restricted delegation only (e.g., ask each voter for a weak order and then delegate each indifference cluster to one peer).
- Minimally ‘fix’ the delegation multi-graph to avoid the problem.
- Move to ‘closest’ delegation multi-graph under which there would be a ‘clear winner’ and elect it (so-called distance rationalisation).

Pairwise Delegation: Two Further Options

Instead of avoiding incompleteness/cycles in the induced preferences, we could develop voting rules specifically for nonstandard preferences (as done, in a different context, by Terzopoulou and Endriss, 2019).

We could also move to a richer model of liquid democracy and obtain rational preferences via opinion diffusion (Christoff and Grossi, 2017):

- Assume each voter has an initial (default) preference order.
- In every round, for each pair, you copy the pairwise ranking of your direct proxy for that pair—if doing so preserves rationality.
- Christoff and Grossi identify conditions under which this stabilises.


Epistemic Social Choice

Suppose there is an objectively ‘correct’ ranking of the alternatives, but each voter only has access to a noisy image of that ranking.

Can we use a voting rule to recover the ‘best’ alternative?

This idea goes back all the way to Condorcet:

**Theorem 1 (Condorcet, 1785)** Suppose a jury of $n$ voters need to select the better of two alternatives and each voter independently makes the correct decision with the same probability $p > \frac{1}{2}$. Then the probability that the simple majority rule returns the correct decision increases monotonically in $n$ and approaches 1 as $n$ goes to infinity.

Effects of Delegation on Epistemic Accuracy

Suppose three voters with accuracies 80%, 75%, 70% choose between two alternatives. Probability of the simple majority rule being right:

\[
\begin{align*}
&0.8 \cdot 0.75 \cdot 0.7 + 0.8 \cdot 0.75 \cdot 0.3 + 0.8 \cdot 0.25 \cdot 0.7 + 0.2 \cdot 0.75 \cdot 0.7 = 0.845 \\
&\quad \text{all are right} \quad \text{exactly two voters are right}
\end{align*}
\]

But if the second voter delegates to the first (most accurate!) voter, then the outcome under the majority rule depends solely on the ballot of that first voter, so the probability of being right goes down to 0.8.

When the accuracies are 80%, 70%, 60%, we get the opposite effect:

\[
0.8 \cdot 0.7 \cdot 0.6 + 0.8 \cdot 0.7 \cdot 0.4 + 0.8 \cdot 0.3 \cdot 0.6 + 0.2 \cdot 0.7 \cdot 0.6 = 0.788 \quad < \quad 0.8
\]

Intuition: Delegation is beneficial for epistemic accuracy (only) when (popular) proxies are significantly better informed than other voters.
Impossibility of Beating Direct Democracy

For $|A| = 2$, Kahng et al. (2018) show that, if voters choose who to delegate to based only on local information, liquid democracy cannot outperform direct democracy in terms of epistemic accuracy.

Assumption: Voters only delegate to friends on a social network who have higher accuracy than they have themselves.

Intuition: For large $n$, many voters may delegate to very few proxies, while voting directly would be better (Condorcet Jury Theorem). But when you are just one of a few voters considering to delegate to a highly accurate proxy, then you should! When voters can only see their local neighbourhood, they are unable to distinguish these situations.

Possibility with Global Coordination

Kahng et al. (2018) also propose a system that can outperform direct democracy. Each voter propose several others they could delegate to. A centralised algorithm then instantiates a delegation graph consistent with these choices such that no casting voter gets too many followers.

The Bigger Picture

Liquid democracy (as in: transitive proxy voting) is but a particularly salient example for a much broader research agenda currently forming:

- Behrens et al. (2014), creators of the *LiquidFeedback* platform, discuss challenges such as the fair *elicitation of proposals*.
- Brill (2018) outlines a broader research agenda for building new theoretical foundations for *participatory decision making*.
- Grandi (2017) reviews research at the interface of social choice theory with *social network analysis* more generally. This includes research on (discrete) *opinion diffusion* on social networks.


M. Brill. Interactive Democracy. AAMAS-2018 (Blue Sky Ideas Track).

Summary

This has been an introduction to some of the research topics currently being considered in the fledging research area of liquid democracy.

What next? Another innovative idea, participatory budgeting.