Homework #4

Deadline: Tuesday, 18 October 2022, 19:00

Exercise 1 (10 points)

Recall that the impossibility direction of the agenda characterisation theorem due to Nehring and Puppe proved in class establishes that all neutral, independent, and monotonic judgment aggregation rules that guarantee consistent and complete outcomes (for a sufficiently rich agenda Φ) must be dictatorships. We might consider weakening the requirement of returning judgment sets that are complete and only ask for outcomes that are deductively closed: judgment set $J \subseteq \Phi$ is *deductively closed* (with respect to Φ) if $J \models \varphi$ implies $\varphi \in J$ for every proposition $\varphi \in \Phi$. The purpose of this exercise is to show that this relaxation of our requirements does not significantly improve the situation. Prove the following theorem:

> *Every propositionwise-unanimous, neutral, independent, and monotonic aggregation rule F that guarantees consistent and deductively closed outcomes for an agenda Φ violating the median property must be an oligarchy.*

Here an *oligarchy* is an aggregation rule F for which there exists a coalition $C^\star \subseteq N$ such that $F(\boldsymbol{J}) = \{\varphi \in \Phi \mid C^\star \subseteq N_\varphi^{\boldsymbol{J}}\}$ for every profile $\boldsymbol{J} \in \mathcal{J}(\Phi)^n$. Thus, a proposition gets accepted by the rule if and only if all oligarchs accept it. Observe that dictatorships and the unanimity rule (i.e., the uniform quota rule with quota $\lambda = n$) are examples for such oligarchies. The *unanimity axiom* is included in our list of assumptions to rule out trivial counterexamples such as the aggregation rule that always returns the empty judgment set.

Hints: Recall that every neutral and independent aggregation rule (for a nontrivial agenda) can be described in terms of a set $\mathcal{W}$ of winning coalitions. Start by establishing some of the structural properties of $\mathcal{W}$, given the assumptions made for our theorem. They will be similar but not identical to the structural properties discussed in class. Then think about what you can say about the intersection of all winning coalitions in $\mathcal{W}$.

Exercise 2 (10 points)

In the context of PB, suppose the preferences of voters depend only on the sum of the costs of the funded projects they truthfully approve of. Demonstrate that the greedy approval mechanism is not strategyproof for such preferences. Then prove that the same mechanism is stratgeyproof for scenarios in which every project has the same cost of 1.