CS2LAP: Logic and Prolog 2000/2001
Sample Solutions for Coursework 2 (Logic)

Question 1.

(a) Natural deduction proof for $(P{\Rightarrow}(Q{\Rightarrow}R)){\Rightarrow}((P \wedge Q){\Rightarrow}R)$:

> $\underline{(P{\Rightarrow}(Q{\Rightarrow}R)){\Rightarrow}((P \wedge Q){\Rightarrow}R)}$
>
> (1) $(P{\Rightarrow}(Q{\Rightarrow}R)){\Rightarrow}((P \wedge Q){\Rightarrow}R)$ from subcomputation
>
> > $\underline{(P \wedge Q){\Rightarrow}R}$
> >
> > (1.1) $P{\Rightarrow}(Q{\Rightarrow}R)$ assumption
> > (1.2) $(P \wedge Q){\Rightarrow}R$ from subcomputation
> >
> > > $\underline{R}$
> > >
> > > (1.2.1) $P \wedge Q$ assumption
> > > (1.2.2) P ($\wedge$E) with (1.2.1)
> > > (1.2.3) Q ($\wedge$E) with (1.2.1)
> > > (1.2.4) $Q{\Rightarrow}R$ modus ponens with (1.2.2) and (1.1)
> > > (1.2.5) R modus ponens with (1.2.3) and (1.2.4)

(b) Natural deduction proof for $(P \wedge \neg P){\Rightarrow}Q$:

> $\underline{(P \wedge \neg P){\Rightarrow}Q}$
>
> (1) $(P \wedge \neg P){\Rightarrow}Q$ from subcomputation
>
> > $\underline{Q}$
> >
> > (1.1) $P \wedge \neg P$ assumption
> > (1.2) $\neg Q{\Rightarrow}P$ from subcomputation
> >
> > > $\underline{P}$
> > >
> > > (1.2.1) $\neg Q$ assumption
> > > (1.2.2) P ($\wedge$E) with (1.1)
> >
> > (1.3) $\neg Q{\Rightarrow}\neg P$ from subcomputation
> >
> > > $\underline{\neg P}$
> > >
> > > (1.3.1) $\neg Q$ assumption
> > > (1.3.2) $\neg P$ ($\wedge$E) with (1.1)
> >
> > (1.4) Q ($\neg$E) with (1.2) and (1.3)

Sample solutions for **Questions 2**, **3**, and **4** are available as Prolog programs from the CS2LAP course web site:

`http://www.dcs.kcl.ac.uk/teaching/units/cs2lap/`