CS2LAP: Logic and Prolog 2000/2001
Sample Solutions for Coursework 3 (Logic)

Question 1.

(a) (i) $\forall x.(dog(x) \Rightarrow animal(x))$

(ii) $\exists x.(child(x) \wedge friend(Fido, x))$

(iii) $(\neg\exists x.(child(x) \wedge dog(x))) \Rightarrow (dog(Fido) \Rightarrow \neg child(Fido))$

(b) (i) Everyone has got a friend.

(ii) There exists a dog all of whose friends are children.

Question 2. In the following goal-directed proofs we delete implications from the data whenever they have been used with the rule for atoms.

(a) We start by rewriting data and goal formula in ready for computation form.

$$\begin{array}{rcl}
P\Rightarrow\bot & \vdash & ((Q\Rightarrow\bot)\Rightarrow P)\Rightarrow Q \\
\mid & \text{rule for } \Rightarrow & \\
P\Rightarrow\bot, (Q\Rightarrow\bot)\Rightarrow P & \vdash & Q \\
\mid & \text{rule for atoms with } \bot & \\
(Q\Rightarrow\bot)\Rightarrow P & \vdash & P \\
\mid & \text{rule for atoms} & \\
 & \vdash & Q\Rightarrow\bot \\
\mid & \text{rule for } \Rightarrow & \\
Q & \vdash & \bot \\
\mid & \text{restart} & \\
Q & \vdash & Q \\
\mid & & \\
\text{success} & &
\end{array}$$

(b) We need to translate $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$ into ready for computation form first: $(P \Rightarrow Q) \wedge Q \Rightarrow P$.

$$Q,\ (P \Rightarrow Q) \wedge Q \Rightarrow P \quad \vdash \quad P$$

rule for atoms

$$Q \quad \vdash \quad (P \Rightarrow Q) \wedge Q$$

rule for $\wedge$

$$Q \quad \vdash \quad P \Rightarrow Q \qquad\qquad Q \quad \vdash \quad Q$$

rule for $\Rightarrow$ success

$$Q,\ P \quad \vdash \quad Q$$

success

Sample solutions for **Questions 3**, **4**, and **5** are available as Prolog programs from the CS2LAP course web site:

`http://www.dcs.kcl.ac.uk/teaching/units/cs2lap/`