CS2LAP: Logic and Prolog 2000/2001
Sample Solutions for Coursework 4 (Logic)

Question 1.

(a) Bound variables in (1): x, y

(b) Free variables in (3): y, x

(c) Bound terms in (2): z, $f(z)$

(d) Free terms in (2): x, y, $g(x,z)$

(e) Free subformulas in (2): $A(x,y)$, $B(x,g(x,z))$, $A(x,y)\Rightarrow B(x,g(x,z))$

(f) Subformulas in the scope of $\exists x$ in (3): $A(f(x))$, $A(y)$, $A(f(x))\Rightarrow A(y)$

(g) Formula (1) is the only sentence (it has no free variables).

Question 2.

(a)

$$\forall x.(A(x)\Rightarrow B(x)) \quad \vdash \quad (\forall x.A(x))\Rightarrow(\forall x.B(x))$$

rule for $\Rightarrow$

$$\forall x.(A(x)\Rightarrow B(x)),\ \forall x.A(x) \quad \vdash \quad \forall x.B(x)$$

Skolemize goal

$$\forall x.(A(x)\Rightarrow B(x)),\ \forall x.A(x) \quad \vdash \quad B(k)$$

rewrite data with free variables

$$\forall x.(A(x)\Rightarrow B(x)),\ \forall x.A(x),\ A(u)\Rightarrow B(u),\ A(v) \quad \vdash \quad B(k)$$

substitution $\{u=k, v=k\}$

$$\forall x.(A(x)\Rightarrow B(x)),\ \forall x.A(x),\ A(k)\Rightarrow B(k),\ A(k) \quad \vdash \quad B(k)$$

rule for atoms

$$\forall x.(A(x)\Rightarrow B(x)),\ \forall x.A(x),\ A(k)\Rightarrow B(k),\ A(k) \quad \vdash \quad A(k)$$

success

(b)

$$\vdash \quad \exists x.\forall y.(A(x) \Rightarrow A(y))$$

rewrite goal with free variable

$$\vdash \quad \forall y.(A(u) \Rightarrow A(y))$$

Skolemize goal

$$\vdash \quad A(u) \Rightarrow A(f(u))$$

rule for $\Rightarrow$

$$A(u) \quad \vdash \quad A(f(u))$$

Note that we cannot unify u and $f(u)$ here. We use the restart rule with the original goal instead.

$$\vdash \quad \exists x.\forall y.(A(x) \Rightarrow A(y))$$

rewrite goal with new free variable

$$A(u) \quad \vdash \quad \forall y.(A(v) \Rightarrow A(y))$$

Skolemize goal

$$A(u) \quad \vdash \quad A(v) \Rightarrow A(g(v))$$

rule for $\Rightarrow$

$$A(u),\, A(v) \quad \vdash \quad A(g(v))$$

substitution $\{u = g(v)\}$

$$A(g(v)),\, A(v) \quad \vdash \quad A(g(v))$$

success

Sample solutions for **Questions 3**, **4**, and **5** are available as Prolog programs from the CS2LAP course web site:

`http://www.dcs.kcl.ac.uk/teaching/units/cs2lap/`