Computational Social Choice
Session 3: Judgment Aggregation

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Introduction

Opening Example

Three agents hold different views on the truth of the propositions p, q, and $p \to q$ (e.g., p might stand for "the temperature is below 16°C" and q for "we should switch off the air conditioning").

	p	$p \to q$	q
Agent 1:	Yes	Yes	Yes
Agent 2:	Yes	No	No
Agent 3:	No	Yes	No

What should be the *collective decision* of the group?

Plan for Today

This will be an introduction to the theory of judgment aggregation.

Main topics to be covered:

- Part 1: Basic Theory
  - more on paradoxes, formal framework
  - axiomatic method and a basic impossibility theorem
- Part 2: Specific Aggregation Rules
- Part 3: Agenda Characterisation
- Part 4: Other Topics
  - strategic considerations
  - applications

U. Endriss. Judgment Aggregation. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A.D. Procaccia (eds.), *Handbook of Computational Social Choice*. CUP, 2015.

Basic Theory

The Doctrinal Paradox

Suppose a court with three judges is considering a case in contract law. Legal doctrine stipulates that the defendant is *liable* (r) *iff* the contract was *valid* (p) and it has been *breached* (q): $r \leftrightarrow p \wedge q$.

	p	q	r
Judge 1:	Yes	Yes	Yes
Judge 2:	No	Yes	No
Judge 3:	Yes	No	No
Majority:	Yes	Yes	No

Paradox: Taking majority decisions on the *premises* (p and q) and then inferring the conclusion (r) yields a different result from taking a majority decision on the *conclusion* (r) directly.

L.A. Kornhauser and L.G. Sager. The One and the Many: Adjudication in Collegial Courts. *California Law Review*, 81(1):1–59, 1993.

The Discursive Dilemma

Our judges were expressing judgements on *atoms* (p, q, r) and consistency of a judgement set was evaluated wrt. an *integrity constraint* ($r \leftrightarrow p \wedge q$).

Alternatively, we could allow judgements on *compound formulas*. Examples:

	p	q	$p \wedge q$
Judge 1:	Yes	Yes	Yes
Judge 2:	No	Yes	No
Judge 3:	Yes	No	No
Majority:	Yes	Yes	No

	p	q	$r \leftrightarrow p \wedge q$	r
Judge 1:	Yes	Yes	Yes	Yes
Judge 2:	No	Yes	Yes	No
Judge 3:	Yes	No	Yes	No
Majority:	Yes	Yes	Yes	No

From now on we will work with a framework without integrity constraints ("legal doctrines"), where all inter-relations between propositions stem from the logical structure of those propositions themselves.

In the philosophical literature, the term *doctrinal paradox* is reserved for the first version of our paradox, and the more general term *discursive dilemma* is used when there is no external "doctrine" that is responsible for the problem.

Why Paradox?

Again, what's paradoxical about our example?

	p	q	$p \wedge q$
Judge 1:	Yes	Yes	Yes
Judge 2:	No	Yes	No
Judge 3:	Yes	No	No
Majority:	Yes	Yes	No

Explanation 1: Two seemingly reasonable methods of aggregation, the *premise-based procedure* and the *conclusion-based procedure*, produce different outcomes.

Explanation 2: Each individual judgment set is logically consistent, but applying the seemingly reasonable *majority rule* to all propositions yields a collective judgment set that is inconsistent.

Formal Framework

Notation: Let $\sim\varphi := \varphi'$ if $\varphi = \neg\varphi'$ and let $\sim\varphi := \neg\varphi$ otherwise.

An *agenda* Φ is a finite nonempty set of propositional formulas (w/o double negation) closed under complementation: $\varphi \in \Phi \;\Rightarrow\; \sim\varphi \in \Phi$.

A *judgment set* J on an agenda Φ is a subset of Φ. We call J:

- *complete* if $\varphi \in J$ or $\sim\varphi \in J$ for all $\varphi \in \Phi$
- *complement-free* if $\varphi \not\in J$ or $\sim\varphi \not\in J$ for all $\varphi \in \Phi$
- *consistent* if there exists an assignment satisfying all $\varphi \in J$

Let $\mathcal{J}(\Phi)$ be the set of all complete and consistent subsets of Φ.

Now a finite set of *individuals* $\mathcal{N} = \{1, \ldots, n\}$, with $n \geqslant 2$, express judgments on the formulas in Φ, producing a *profile* $\boldsymbol{J} = (J_1, \ldots, J_n)$.

An *aggregation procedure* for an agenda Φ and a set of n individuals is a function mapping a profile of complete and consistent individual judgment sets to a single collective judgment set: $F : \mathcal{J}(\Phi)^n \to 2^\Phi$.

Example: Majority Rule

Suppose three agents ($\mathcal{N} = \{1, 2, 3\}$) express judgments on the propositions in the agenda $\Phi = \{p, \neg p, q, \neg q, p \vee q, \neg(p \vee q)\}$.

For simplicity, we only show the positive formulas in our tables:

	p	q	$p \vee q$	
Agent 1:	Yes	No	Yes	$J_1 = \{p, \neg q, p \vee q\}$
Agent 2:	Yes	Yes	Yes	$J_2 = \{p, q, p \vee q\}$
Agent 3:	No	No	No	$J_3 = \{\neg p, \neg q, \neg(p \vee q)\}$

The (strict) *majority rule* F_{maj} takes a (complete and consistent) profile and returns the set of propositions accepted by $> \frac{n}{2}$ agents.

In our example: $F_{\text{maj}}(\boldsymbol{J}) = \{p, \neg q, p \vee q\}$ [complete and consistent!]

In general, F_{maj} only ensures completeness and complement-freeness [and completeness only in case n is odd].

Example: Embedding Preference Aggregation

In *preference aggregation*, individuals express preferences (linear orders) over a set of alternatives $\mathcal{X}$ and we need to find a collective preference.

We can embed this into JA (suppose $\mathcal{X} = \{A, B, C\}$):

- Take atomic propositions to be $[A \succ A]$, $[A \succ B]$, ...
- Suppose all individuals accept these propositions:
  - Irreflexivity: $\neg[A \succ A]$, $\neg[B \succ B]$, $\neg[C \succ C]$
  - Completeness: $[A \succ B] \vee [B \succ A]$ etc.
  - Transitivity: $[A \succ B] \wedge [B \succ C] \rightarrow [A \succ C]$, etc.

Then the famous *Condorcet paradox* corresponds to this example in JA:

	$[A \succ B]$	$[A \succ C]$	$[B \succ C]$	*corresponding order*
Agent 1:	Yes	Yes	Yes	$A \succ B \succ C$
Agent 2:	No	No	Yes	$B \succ C \succ A$
Agent 3:	Yes	No	No	$C \succ A \succ B$
Majority:	Yes	No	Yes	*not a linear order*

Axioms

What makes for a "good" aggregation procedure F? The following *axioms* all express intuitively appealing (yet, debatable) properties:

- *Anonymity:* Treat all individuals symmetrically!

  Formally: for any profile $\boldsymbol{J}$ and any permutation $\pi : \mathcal{N} \to \mathcal{N}$ we have $F(J_1, \ldots, J_n) = F(J_{\pi(1)}, \ldots, J_{\pi(n)})$.

- *Neutrality:* Treat all propositions symmetrically!

  Formally: for any φ, ψ in the agenda Φ and any profile $\boldsymbol{J}$, if for all $i \in \mathcal{N}$ we have $\varphi \in J_i \Leftrightarrow \psi \in J_i$, then $\varphi \in F(\boldsymbol{J}) \Leftrightarrow \psi \in F(\boldsymbol{J})$.

- *Independence:* Only the "pattern of acceptance" should matter!

  Formally: for any φ in the agenda Φ and any profiles $\boldsymbol{J}$ and $\boldsymbol{J'}$, if $\varphi \in J_i \Leftrightarrow \varphi \in J'_i$ for all $i \in \mathcal{N}$, then $\varphi \in F(\boldsymbol{J}) \Leftrightarrow \varphi \in F(\boldsymbol{J'})$.

Observe that the *majority rule* satisfies all of these axioms.

(But so do some other procedures! Can you think of some examples?)

Impossibility Theorem

We have seen that the majority rule is *not consistent*. Is there another "reasonable" aggregation procedure that does not have this problem? *Surprisingly, no!* (at least not for certain agendas)

Theorem 1 (List and Pettit, 2002) *No judgment aggregation procedure for an agenda Φ with $\{p, q, p \wedge q\} \subseteq \Phi$ that satisfies the axioms of anonymity, neutrality, and independence will always return a collective judgment set that is complete and consistent.*

Remark 1: Note that the theorem requires $|\mathcal{N}| > 1$.

Remark 2: Similar impossibilities arise for other agendas with some minimal structural richness. To be discussed later on.

C. List and P. Pettit. Aggregating Sets of Judgments: An Impossibility Result. *Economics and Philosophy*, 18(1):89–110, 2002.

Proof: Part 1

Let $N_\varphi^{\boldsymbol{J}}$ be the set of individuals who accept formula φ in profile $\boldsymbol{J}$.

Let F be any aggregator that is independent, anonymous, and neutral.

We observe:

- Due to *independence*, whether $\varphi \in F(\boldsymbol{J})$ only depends on $N_\varphi^{\boldsymbol{J}}$.
- Then, by *anonymity*, whether $\varphi \in F(\boldsymbol{J})$ only depends on $|N_\varphi^{\boldsymbol{J}}|$.
- Finally, due to *neutrality*, the manner in which $\varphi \in F(\boldsymbol{J})$ depends on $|N_\varphi^{\boldsymbol{J}}|$ must itself *not* depend on φ.

Thus: if φ and ψ are accepted by the same number of individuals, then we must either accept both of them or reject both of them.

Proof: Part 2

Recall: For all $\varphi, \psi \in \Phi$, if $|N_\varphi^{\boldsymbol{J}}| = |N_\psi^{\boldsymbol{J}}|$, then $\varphi \in F(\boldsymbol{J}) \Leftrightarrow \psi \in F(\boldsymbol{J})$.

First, suppose the number n of individuals is *odd* (and $n > 1$).

Consider a profile $\boldsymbol{J}$ where $\frac{n-1}{2}$ individuals accept p and q; one each accept exactly one of p and q; and $\frac{n-3}{2}$ accept neither p nor q.
That is: $|N_p^{\boldsymbol{J}}| = |N_q^{\boldsymbol{J}}| = |N_{\neg(p \wedge q)}^{\boldsymbol{J}}|$. Then:

- Accepting all three formulas contradicts consistency. ✓
- But if we accept none, completeness forces us to accept their complements, which also contradicts consistency. ✓

If n is *even*, we can get our impossibility even without having to make any assumptions regarding the structure of the agenda:

Consider a profile $\boldsymbol{J}$ with $|N_p^{\boldsymbol{J}}| = |N_{\neg p}^{\boldsymbol{J}}|$. Then:

- Accepting both contradicts consistency. ✓
- Accepting neither contradicts completeness. ✓

Change of Perspective

Does the impossibility theorem mean that all hope is lost? *No.*

- We could analyse in more depth for *what agendas* this problem can actually occur. And if it can, we could analyse *how to detect* such a situation. We will follow this route a little later.
- We could argue that it is ok to *weaken those axioms:*
  - *Anonymity:* maybe some agents are smarter than others?
  - *Neutrality:* maybe it is actually ok to treat, say, atomic propositions differently from conjunctions?
  - *Independence:* there *are* logical dependencies between propositions; so why not allow them to affect aggregation?

Specific Aggregation Rules

Specific Aggregation Rules

Next, we look into some practical aggregators that circumvent the noted impossibility. Thus, they all violate at least one of the axioms.

Quota Rules

A *quota rule* F_q is defined by a function $q : \Phi \rightarrow \{0, 1, \ldots, n{+}1\}$:

$$F_q(\boldsymbol{J}) \quad = \quad \{\varphi \in \Phi \mid |N_\varphi^{\boldsymbol{J}}| \geqslant q(\varphi)\}$$

A quota rule F_q is called *uniform* if q maps any given formula to the same number k. Examples:

- The *unanimous rule* F_n accepts φ *iff* everyone does.
- The *constant rule* F_0 (F_{n+1}) accepts all (no) formulas.
- The *(strict) majority rule* F_{maj} is the quota rule with $q = \lceil \frac{n+1}{2} \rceil$.
- The *weak majority rule* is the quota rule with $q = \lceil \frac{n}{2} \rceil$.

Observe that for *odd* n the majority rule and the weak majority rule coincide. For *even* n they differ (and only the weak one is complete).

Quota Rules with a High Quota

Clearly, a (uniform) quota rule with a sufficiently high quota will be consistent. Dietrich and List (2007) give lower bounds for the quota to ensure consistency as a function of n and the size of the largest *minimally inconsistent subset* of the agenda Φ. Example:

> Let $\Phi = \{p, \neg p, q, \neg q, p \wedge q, \neg(p \wedge q)\}$. The largest mi-subset is $\{p, q, \neg(p \wedge q)\}$. Any quota $> \frac{2}{3}$ will ensure consistency.

But: We (may) lose completeness of the collective judgment set.

F. Dietrich and C. List. Judgment Aggregation by Quota Rules: Majority Voting Generalized. *Journal of Theoretical Politics*, 19(4):391–424, 2007.

Characterisation of Quota Rules

Quota rules are nice to demonstrate the axiomatic method ...

One more axiom:

- *Monotonicity:* If an accepted proposition gets additional support, then we should continue to accept it!

  Formally: for any $\varphi \in \Phi$ and profiles $\boldsymbol{J}, \boldsymbol{J'}$, if $\varphi \in J'_{i^\star} \setminus J_{i^\star}$ for some $i^\star$ and $J_i = J'_i$ for all $i \neq i^\star$, then $\varphi \in F(\boldsymbol{J}) \Rightarrow \varphi \in F(\boldsymbol{J'})$.

We can now *characterise* the class of quota rules:

Proposition 2 (Dietrich and List, 2007) *An aggregation procedure is anonymous, independent and monotonic iff it is a quota rule.*

F. Dietrich and C. List. Judgment Aggregation by Quota Rules: Majority Voting Generalized. *Journal of Theoretical Politics*, 19(4)391–424, 2007.

Proof

Claim: *anonymous* + *independent* + *monotonic* $\Leftrightarrow$ *quota rule*

Clearly, any quota rule has these properties (right-to-left).

For the other direction (using the same technique as before):

- Independence means that acceptance of φ only depends on $N_\varphi^{\boldsymbol{J}}$.
- Anonymity means that, in fact, it only depends on $|N_\varphi^{\boldsymbol{J}}|$.
- Monotonicity means that acceptance of φ cannot turn to rejection as additional individuals accept φ.

Hence, it must be a quota rule. ✓

Reminder: $N_\varphi^{\boldsymbol{J}}$ is the set of individuals who accept φ in profile $\boldsymbol{J}$.

More Characterisations

Clearly, a quota rule F_q is uniform *iff* it is neutral. Thus:

Corollary 3 *An aggregation procedure is anonymous, neutral, independent and monotonic ($=$ ANIM) iff it is a uniform quota rule.*

Now consider a uniform quota rule F_q with quota q. Two observations:

- For F_q to be *complete*, we need $q \leqslant \max_{0 \leqslant x \leqslant n}(x, n-x) \Rightarrow q \leqslant \lceil \frac{n}{2} \rceil$.
- For F_q to be *compl.-free*, we need $q > \min_{0 \leqslant x \leqslant n}(x, n-x) \Rightarrow q > \lfloor \frac{n}{2} \rfloor$.

For n *even*, no such q exists. Thus:

Proposition 4 *For n even, no aggregation procedure is ANIM, complete and complement-free.*

For n *odd*, such a q does exist, namely $q = \lceil \frac{n}{2} \rceil = \frac{n+1}{2}$. Thus:

Proposition 5 *For n odd, an aggregation procedure is ANIM, complete and complement-free iff it is the (strict) majority rule.*

The Premise-Based Procedure

Suppose we *can* divide the agenda into *premises* and *conclusions* (i.e., we are willing to give up *neutrality*):

$$\Phi \quad = \quad \Phi_p \uplus \Phi_c$$

The *premise-based procedure* PBP for Φ_p and Φ_c is this function:

$$\begin{aligned} \mathrm{PBP}(\boldsymbol{J}) \quad &= \quad \Delta \cup \{\varphi \in \Phi_c \mid \Delta \models \varphi\}, \\ & \quad\quad \text{where } \Delta = \{\varphi \in \Phi_p \mid |\{i \mid \varphi \in J_i\}| > \frac{n}{2}\} \end{aligned}$$

If we assume that

- the set of *premises* is the set of *literals* in the agenda,
- the agenda Φ is *closed under propositional letters*, and
- the number n of individuals is *odd*,

then $\mathrm{PBP}(\boldsymbol{J})$ will always be *consistent* and *complete*.

Example: Violation of Propositionwise Unanimity

Consider the following basic axiom:

- *Propositionwise Unanimity*: $\varphi \in J_i$ for all $i \in \mathcal{N} \Rightarrow \varphi \in F(\boldsymbol{J})$.

  Unanimous acceptance implies collective acceptance!

Curiously, the premise-based procedure violates this form of unanimity:

	p	q	r	$p \vee q \vee r$
Agent 1:	Yes	No	No	Yes
Agent 2:	No	Yes	No	Yes
Agent 3:	No	No	Yes	Yes
PBP:	No	No	No	No

Distance-Based Procedures

A general approach to designing aggregation procedures is to fix a notion of *distance* between judgments sets and then to use it to define what it means for a judgment set to be *closest* to the input profile amongst all consistent judgment sets and to then return that set.

The most widely used distance is the *Hamming distance:*

$$H(J, J') = \frac{1}{2} \cdot |J \setminus J' \cup J' \setminus J|$$

There are several ways of turning this into an aggregator . . .

G. Pigozzi. Belief Merging and the Discursive Dilemma: An Argument-based Account of Paradoxes of Judgment. *Synthese*, 152(2):285–298, 2006.

M.K. Miller and D. Osherson. Methods for Distance-based Judgment Aggregation. *Social Choice and Welfare*, 32(4):575–601, 2009.

The Standard Distance-Based Procedure

The standard *distance-based procedure* is defined as follows:

$$\mathrm{DBP}(\boldsymbol{J}) \quad = \quad \underset{J \in \mathcal{J}(\Phi)}{\operatorname{argmin}} \sum_{i=1}^{n} H(J, J_i)$$

Recall that $\mathcal{J}(\Phi)$ is the set of all complete and consistent judgment sets. So the DBP is complete and consistent *by definition*.

Some remarks:

- The DBP may return a set of tied winners ("*irresolute*").
- It is *not independent*.
- If the majority winner is consistent, then it is also the DBP-winner.

The DBP is based on the same principle as the *Kemeny rule*.

Another Distance-Based Procedure

Rather than selecting a consistent outcome that is closest to the profile, we may select a consistent outcome that is closest to the (possibly inconsistent) majority outcome:

$$\mathrm{DBP}'(\boldsymbol{J}) \quad = \quad \operatorname*{argmin}_{J \in \mathcal{J}(\Phi)} H(F_{\mathsf{maj}}(\boldsymbol{J}), J)$$

This method is based on the same principle as the *Slater rule*.

Winner Determination

A *disadvantage* of the DBP is its high complexity. Consider the *winner determination* problem, asking whether a given partial judgement set can be extended to a winning judgment set for a given profile.

Theorem 6 *Winner determination for the* DBP *is* Θ_2^p*-complete.*

Proof: Omitted. [Θ_2^p is also known as "parallel access to NP"]

Compare this to the other aggregation procedures we have discussed:

Fact 7 *Winner determination for any quota rule* F_q *is in P.*

Proposition 8 *Winner determination for the* PBP *is in P.*

Proof: counting (for premises) + model checking (for conclusions) ✓

U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.

Agenda Characterisation

Agenda Characterisation

Recall our *impossibility theorem:* no consistent aggregator is independent, neutral, and anonymous for agendas $\Phi \supseteq \{p, q, p \wedge q\}$.

More interesting question:

- For which *class of agendas* is *consistent aggregation* (im)possible?

We will give several answers to this generic question . . .

Remark: Note that the characterisation results we have seen so far (e.g., axiomatisation of the majority rule) are rather different. They don't involve consistency (i.e., they don't involve any logic).

Consistent Aggregation under the Majority Rule

Previously we saw that the *majority rule* can produce an inconsistent outcome for *some* (not all) profiles based on agendas $\Phi \supseteq \{p, q, p \wedge q\}$. How can we *characterise* the class of agendas with this problem?

A set of formulas Φ satisfies the *median property* if every inconsistent subset of Φ does itself have an inconsistent subset of size $\leqslant 2$.

Lemma 9 (Nehring and Puppe, 2007) *Let $n \geqslant 3$. The majority rule is consistent for a given agenda Φ iff Φ has the median property.*

Remark: Note how $\{p, \neg p, q, \neg q, p \wedge q, \neg(p \wedge q)\}$ violates the MP.

K. Nehring and C. Puppe. The Structure of Strategy-proof Social Choice. Part I: General Characterization and Possibility Results on Median Space. *Journal of Economic Theory*, 135(1):269–305, 2007.

Proof

Claim: Φ is *safe* [$F_{\text{maj}}(\boldsymbol{J})$ is consistent] $\Leftrightarrow$ Φ has the median property

($\Leftarrow$) Let Φ be an agenda with the median property. Now assume that there exists an admissible profile $\boldsymbol{J}$ such that $F_{\text{maj}}(\boldsymbol{J})$ is *not* consistent.

- $\leadsto$ There exists an inconsistent set $\{\varphi, \psi\} \subseteq F_{\text{maj}}(\boldsymbol{J})$.
- $\leadsto$ Each of φ and ψ must have been accepted by a strict majority.
- $\leadsto$ One individual must have accepted both φ and ψ.
- $\leadsto$ Contradiction (individual judgment sets must be consistent). ✓

($\Rightarrow$) Let Φ be an agenda that violates the median property, i.e., there exists a minimally inconsistent set $\Delta = \{\varphi_1, \ldots, \varphi_k\} \subseteq \Phi$ with $k > 2$.

Consider the profile $\boldsymbol{J}$, in which individual i accepts all formulas in Δ except for $\varphi_{1+(i \bmod 3)}$. Note that $\mathcal{J}$ is consistent. But the majority rule will accept all formulas in Δ, i.e., $F_{\text{maj}}(\boldsymbol{J})$ is inconsistent. ✓

Complexity of Safety of the Agenda

Deciding whether a given agenda is safe for the majority rule is located at the second level of the polynomial hierarchy.

Proving this involves the following lemma:

Lemma 10 (Endriss et al., 2012) *Deciding whether a given agenda has the median property is Π_2^p-complete.*

Proof: Omitted.

Recall that $\Pi_2^p = \text{coNP}^{\text{NP}}$ is the class of problems for which we can verify a certificate for a negative answer in polynomial time if we have access to an NP oracle. A typical problem in the class is deciding truth of formulas of the form $\forall \boldsymbol{x} \exists \boldsymbol{y} \varphi(\boldsymbol{x}, \boldsymbol{y})$. So: very hard.

U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.

Agenda Characterisation for Classes of Rules

Instead of a single aggregator, suppose we are interested in a *class of aggregators*, possibly determined by a set of *axioms*. We might ask:

- *Possibility:* Does there exist an aggregator meeting certain axioms that will be consistent for any agenda with a given property? (think of an economist looking for a rule meeting certain axioms)
- *Safety:* Will every aggregator meeting certain axioms be consistent for any agenda with a given property? (think of a multiagnet system about which we have only partial knowlege)

K. Nehring and C. Puppe. Abstract Arrovian Aggregation. *Journal of Economic Theory*, 145(2):467–494, 2010.

C. List and C. Puppe. Judgment Aggregation: A Survey. In: *Handbook of Rational and Social Choice*, Oxford University Press, 2009.

U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.

Other Topics

Example: Strategic Manipulation

Suppose we use the premise-based procedure:

	p	q	$p \vee q$
Agent 1:	No	No	No
Agent 2:	Yes	No	Yes
Agent 3:	No	Yes	**Yes**
PBP:	No	No	No

If agent 3 only cares about the conclusion $p \vee q$, she could *manipulate* the aggregation by claiming to believe that p is true.

Strategic Manipulation

Note that in pure JA, we cannot talk about strategic behaviour, as there is no notion of preference. We need to add one! *How?*

This is still underexplored territory. Main definition in use so far:

- Your true judgment set is your most preferred outcome.
- The closer an outcome to your true judgment set, in terms of the *Hamming distance*, the more you prefer that outcome.

Thus: Agent i with true judgment set J_i *prefers* J to J' ($J \succ_i J'$) *iff*

$$H(J_i, J) \quad < \quad H(J_i, J').$$

Now i has an *incentive to manipulate* if $F(\boldsymbol{J}_{-i}, J) \succ_i F(\boldsymbol{J})$ for some $J \neq J_i$.

We say that F is *immune to manipulation* if it never provides an incentive to manipulate to any of the agents.

Recall: $H(J, J') = \frac{1}{2} \cdot |J \setminus J' \cup J' \setminus J|$

Immunity to Manipulation

Some aggregators are immune to manipulation:

Proposition 11 (Dietrich and List, 2007) *An aggregator is immune to manipulation iff it is both independent and monotonic.*

Proof: We only sketch the right-to-left direction. *Independence* means that we can consider each formula in isolation. But then *monotonicity* just means that it is never against an agent's interest to accept a formula she holds true. ✓

But note that the class of independent and monotonic aggregators is *not that attractive* (dictatorships, inconsistent majority rule, ...).

F. Dietrich and C. List. Strategy-proof Judgment Aggregation. *Economics and Philosophy*, 23(3):269–300, 2007.

Hardenss of Manipulation

When there is no immunity from manipulation, we at least would like manipulation to be computationally hard.

$\textsc{Manipulable}(F)$
Instance: Agenda Φ, profile $\boldsymbol{J} \in \mathcal{J}(\Phi)^n$, agent $i \in \mathcal{N}$.
Question: Is there a $J \in \mathcal{J}(\Phi)$ such that $F(\boldsymbol{J}_{-i}, J) \succ_i F(\boldsymbol{J})$?

Good news:

Proposition 12 $\textsc{Manipulable}(\text{PBP})$ *is NP-complete.*

Proof omitted (works by reduction from $\textsc{Sat}$). Only case we know where winner determination is easy and manipulation is hard.

U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.

Applications

Some recent work has at least suggested possible directions for using judgment aggregation techniques in applications. Examples:

- Collective decision making in multiagent systems
- Ontology merging on the Semantic Web
- Aggregating crowdsourced data (e.g., for computational linguistics)

M. Slavkovik. Judgment Aggregation for Multiagent Systems. PhD thesis, University of Luxembourg, 2012.

D. Porello and U. Endriss. Ontology Merging as Social Choice: Judgment Aggregation under the Open World Assumption. *J. Logic and Computation*. In press.

U. Endriss and R. Fernández. Collective Annotation of Linguistic Resources: Basic Principles and a Formal Model. Proc. ACL-2013.

Conclusion

Summary: Judgment Aggregation

We have covered the basics of the theory of judgment aggregation:

- Questions first raised by real-world problems in legal scholarship with philosophical ramifications (*doctrinal paradox*); later formalised in the style of *social choice theory*
- *Formal basics:* axioms, List-Pettit impossibility, characterisations
- *Rules:* based on quota, premises, distances
- *Agenda characterisations:* possibility and safety, complexity
- *Strategic considerations:* manipulation, complexity
- *Application:* MAS, ontology merging, crowdsourcing

The full potential of judgment aggregation for applications in computer science is yet to be explored.

Literature: Judgment Aggregation

The tutorial paper by List (2012) is an easy-going introduction to JA, focussing on how to circumvent the original impossibility theorem.

My own chapter is more technical and covers all the material of today.

C. List. The Theory of Judgment Aggregation: An Introductory Review. *Synthese*, 187(1):179–207, 2012.

U. Endriss. Judgment Aggregation. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A.D. Procaccia (eds.), *Handbook of Computational Social Choice*. CUP, 2015.

End of Course

This has been an introduction to *computational social choice*.

We have reviewed material from three areas:

- *fair allocation of resources*
- *voting theory*
- *judgment aggregation*

Slides and main references will remain available on the course website:

http://www.illc.uva.nl/~ulle/teaching/fair-2014/

Or visit the website for my Amsterdam course for more material:

http://www.illc.uva.nl/~ulle/teaching/comsoc/