Homework #5

Deadline: Tuesday, 13 May 2025, 13:00

Submit your solutions for (up to) three of the following four exercises. If you solve all four, we will consult a random number generator to decide which three to look at and grade.

Exercise 1 (10 points)

Vickrey auctions are second-price sealed-bid auctions. We have seen the advantages of using second-price rather than first-price auctions in class. Maybe we can get further improvements by introducing a third-price auction?

(a) Describe how such a third-price sealed-bid auction would work.

(b) What would be a good bidding strategy for this type of auction?

(c) Recall that the dominant bidding strategy for private-value Vickrey auctions is to bid your true valuation. Is there a dominant strategy for third-price auctions?

Exercise 2 (10 points)

Consider the following auction design problem. We want to sell k different goods, called $\alpha_1, \ldots, \alpha_k$, to n bidders (with $k < n$). Everyone agrees that the value of α_1 is exactly twice the value of α_2, that the value of α_2 is exactly twice the value of α_3, and so forth. But the bidders might disagree on the absolute values of the goods. Thus, we can fully describe the valuation of a bidder for all items by means of a single (nonnegative) number, her valuation for α_1. A bid consists of such a number. We are going to allocate α_1 to the highest bidder, α_2 to the second highest bidder, and so forth. Ties are broken in favour of bidders submitting their bids early (and we assume that no two bidders can bid at exactly the same time). Finally, the prices to be paid are determined as follows: For all $\ell \leqslant k$, the bidder receiving item α_ℓ must pay $\frac{1}{2^{\ell-1}}$ of the price corresponding to the next highest bid. For example, the highest bidder obtains item α_1 and pays the second highest bid; the second highest bidder gets α_2 and pays half of the third highest bid; and the third highest bidder gets α_3 and pays one quarter of the fourth highest bid.

Is the mechanism described above incentive-compatible? Justify your answer.

Hint: The issue of interest here has nothing to do with tie-breaking, so in your answer, if you find it helpful, you may assume that all bidders have mutually distinct valuations and also that all bidders always report mutually distinct valuations.

Exercise 3 (10 points)

Write a program to compute the outcome (allocation and prices) for a combinatorial auction with single-minded bidders under the VCG mechanism. Report on the performance of your algorithm for randomly generated auction instances of varying size.

Keep in mind that implementing such a combinatorial auction solver involves solving several

NP-hard optimisation problems for each auction instance (one to compute the allocation and a further n of them to compute the prices), so a naïve algorithm is unlikely to work well in all cases for somewhat larger problem instances. Your report should include a discussion of the limitations of your algorithm.

Exercise 4 (10 points)

The purpose of this exercise is to demonstrate how we can model an auction as a Bayesian game. Consider the following situation. Two players are bidding in a first-price sealed-bid auction for a single item. The set of permissible bids is restricted to the set of non-negative integers. In case both players make the same bid, the winner is chosen by means of a fair coin toss. Each player has their own private valuation for the item. It is common knowledge that each player's true valuation has been drawn from the uniform probability distribution over the set $\{0, 1, 2\}$, and these two valuations have been drawn independently of each other. Formally define this Bayesian game. Then compute at least one Bayes-Nash equilibrium for the game you defined. Note that, in a minor deviation from the definition given in class, you may need to allow for certain parameters of your game to be infinite.