Logics for Reasoning about Quantum Information: A Dynamic-Epistemic Perspective

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Based on joint work with Alexandru Baltag

Overview:

- **Part 1: Quantum Information**
  - From Bits to Qubits
- **Part 2: A revolution in logic?**
  - Quantum Logic
  - Is Logic Empirical?
- **Part 3: Applications**
  - Teleportation
  - Quantum key distribution

PART 1: Quantum Information

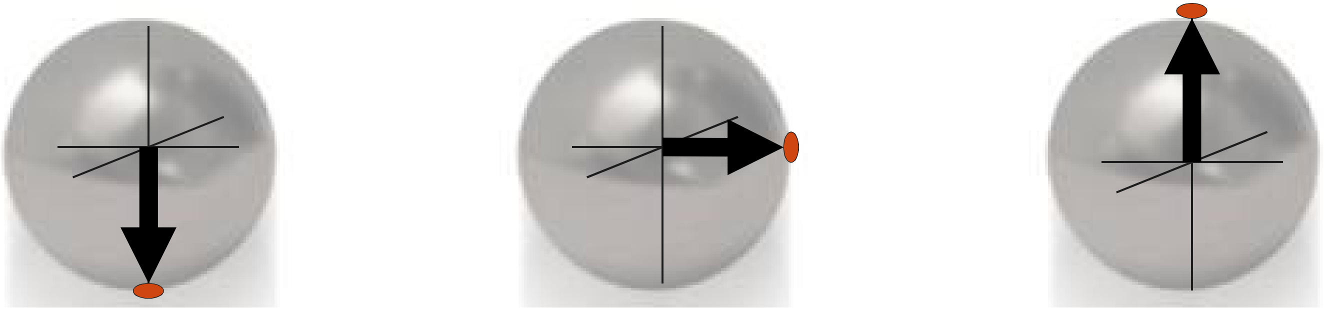
- From Bits to Qubits

Classical information: Bit

- Possible states 0 or 1

Quantum Information: Qubit

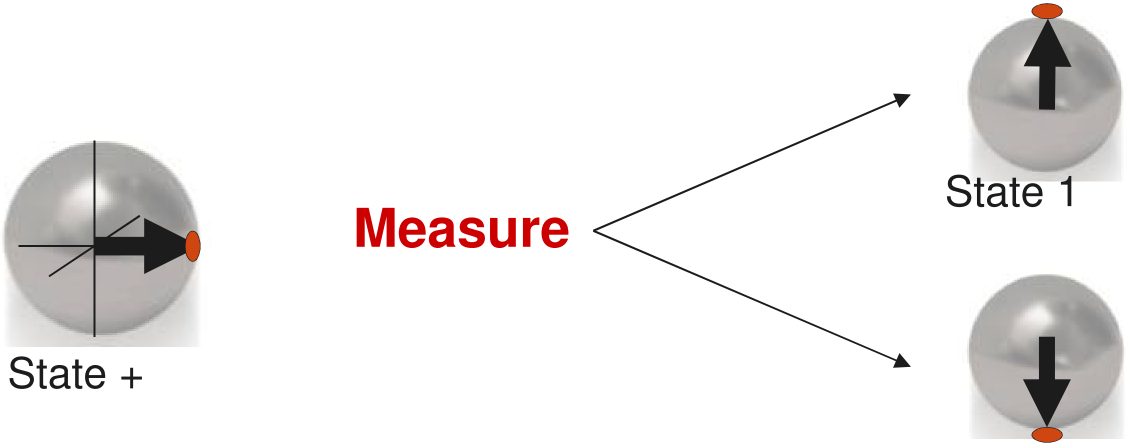
- Possible states 0, 1 and superpositions



State 0 Superposition + State 1

Change of Information: observation

- Observations change the state (“collapse”)



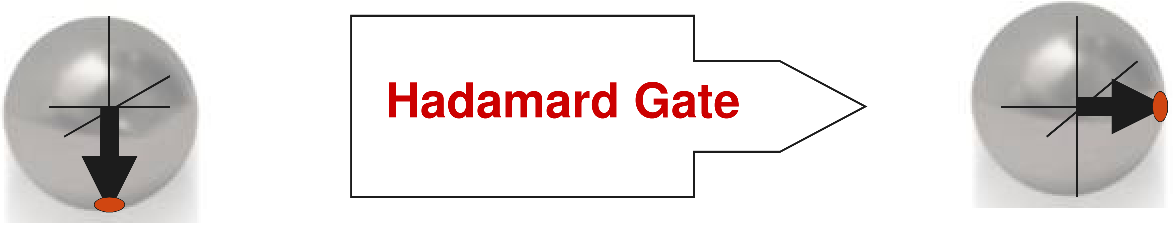
State +

Measure

State 1

State 0

Evolution – Logic gates



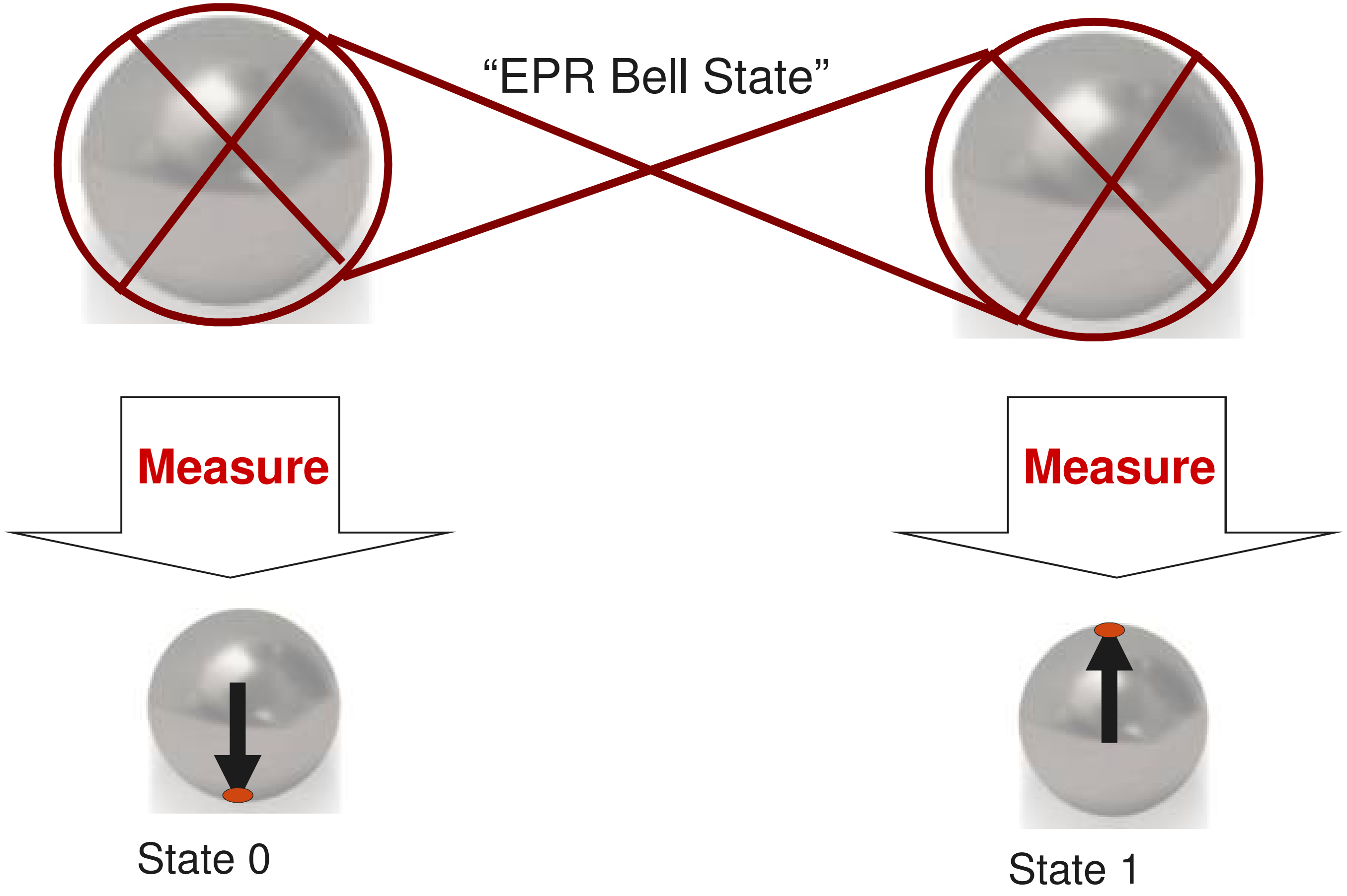
State 0

Hadamard Gate

Superposition +

Entangled Systems

- Non-locality: entanglement makes it possible to manipulate (to access) information from a distance.



“EPR Bell State”

Measure

Measure

State 0

State 1

From Physics to Information (Flow)

- Quantum computing and quantum communication make essential use of these principles of superposition, entanglement and “collapse”.

- **“Quantum computers exploit “quantum weirdness” to perform tasks too complex for classical computers. Because a quantum bit, or “qubit” can register both 0 and 1 *at the same time* (a classical bit can register only one or the other), a quantum computer can perform millions of computations simultaneously” (Seth Lloyd 2005)**

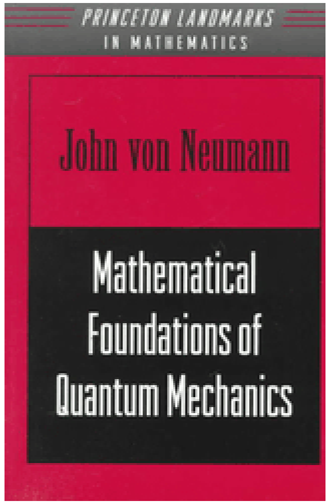
PART 2: A revolution in logic?

Which logic should we use to reason about quantum information?

- Standard quantum logic
- What about classical logic?
- Our approach :
  dynamic quantum logic,
  quantum transition systems
- Is (quantum) logic empirical?

Origin of Quantum Logic

- J. Von Neumann 1932:


PRINCETON LANDMARKS IN MATHEMATICS

John von Neumann

Mathematical Foundations of Quantum Mechanics

“To discover the logical structure one may hope to find in physical theories which, like QM, do not conform to classical logic.”

(Birkhoff and von Neumann 1936)

Traditional Quantum Logic

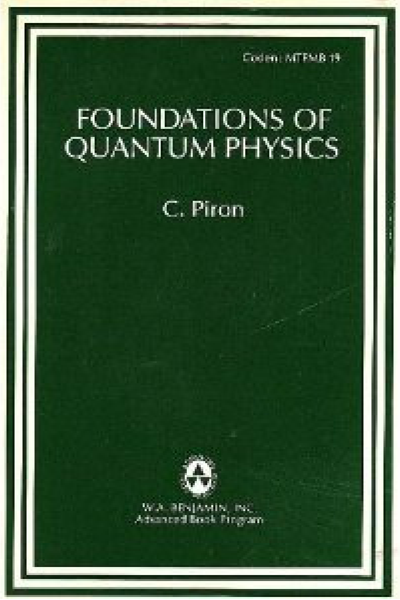
- The beginning Quantum logic looked for the axiomatic structure based on the Hilbert space structure:

- From the 50’s on, we see a shift in focus:
    - quantum logicians looking for the abstract algebraic-logical conditions to describe quantum systems without reference to the Hilbert space structure.

Traditional Quantum Logic

- In the 60-70’s the aim was to give an abstract logical axiomatization and to prove a representation theorem with respect to the Hilbert space structure. (J.M. Jauch en C. Piron)

C. Piron, Geneve



Coden: MTFMB 19

FOUNDATIONS OF
QUANTUM PHYSICS

C. Piron

W.A. BENJAMIN, INC.
Advanced Book Program

Traditional Quantum Logic: basics

- Focus on “testable properties”, these are only the “physically testable or experimental properties” of a quantum system,
- E.g. Property *X* : “The particle is located in the state space region with coordinates *q*0”
- A property can be actual (the corresponding proposition is true), depending on the state of the system.
- Build a Logical calculus of properties: if p is true in state s and q is true in state s, then what about “p or q”, “p and q” ?
- In the traditional view = quantum logic has to give up some basic classical logical principles.

No classical disjunction

- We cannot capture superpositions (of properties) by using the classical disjunction (“OR”)

- $t \models p$ or $t \models q$ $\Longrightarrow$ $t \models (p \,\underline{\vee}\, q)$

  (the other direction fails)

- A quantum system can be in a superposition state in which $(p \underline{\vee} q)$ is true but in which both p is false and q is false.

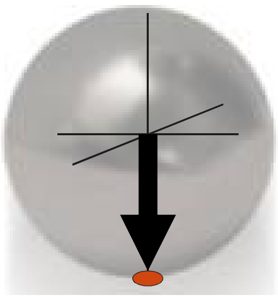
- Consequence: **“Distributivity is a law in classical, not quantum mechanics...”** (Birkhoff & von Neumann)

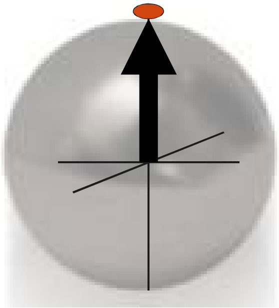
No classical negation

- The orthocomplement of an experimental property is an "experimental property" and is comparable with a strong type of negation.

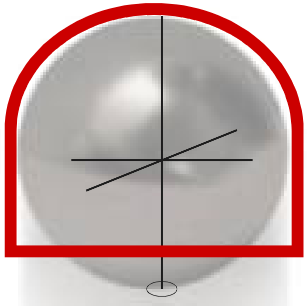
$$t \models \sim p \quad \Longrightarrow \quad t \not\models p$$

(but the other direction fails)


State 0


Orthogonal State

~ 0


Classical Negation

$\neg\, 0$

No classical negation

- The classical negation of testable property is not necessarily itself a “testable property” (and in this logic we only deal with testable properties) :

Example: ***“the system is not in state 0”***

is a non-testable property

Structure of properties

The set of testable properties is not closed under classical negation and classical disjunction.

Traditional quantum logic is the study of the structure of these testable properties:

$(L, \subseteq, \wedge, \underline{\vee}\ , \sim)$

So classical logic is out?

- According to traditional q-logic, we have to give up some classical principles such as distributivity of “and – or”

- Is the principle of bivalence still valid?

- common view: superpositions show that we need an extra truth-value:

Shrödinger’s cat in superposition:

Source: Internet

New approach:

- As I show next, it is not necessary to abandon classical logic. All non-classical properties will emerge as being the consequence of the non-classical flow of quantum information.

- Quantum Logic as a transition system

- **Idea = we characterize the state of a system via the actions that can be performed (successfully) performed in that state.**

In the light of C.Piron’s work:

“What is a quantum object like an electron for example? … The Hilbert space description by the wave function $\psi_t(x)$ is a model of such reality, but is not the electron itself. It gives you all indications about what it is possible to do with such an object,

like a picture of a pipe can give you an idea how to smoke. But please don’t light the painting to smoke the pipe!” C. Piron (1999)

The logic of quantum actions:

- **Model** = possible states and transition-relations for measurements and evolutions (logical gates)

- **How does this work?**

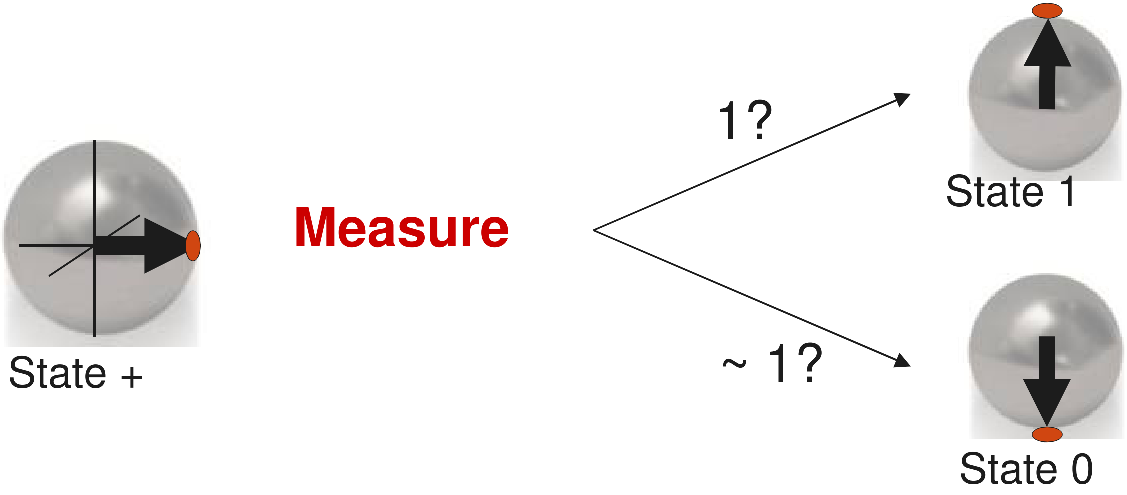
- **Challenge** = to explain all notions that we already saw, like testable property, orthocomplement and superposition, only from the point of view of the actions (measurements and evolutions) that I can perform on the system.

Yes-No-Measurements:

- Measurements can be viewed as a combination of tests
- **Yes-no-measurements:**

  - If Yes, we obtain property P, “ test of P? “

  - If No, we obtain property ~P, “ test of ~P? “



Creation via Measurements

- A quantum system (in contrast to a classical system) can change due to a measurement.

- By measuring a physical property P, the state can change such that the system will have this property AFTER the experiment.

- Due to a measurement, the original state gets lost, there is no reason to assume that the system had property P before the experiment.

Indeterminism

- **Quantum measurements are inherent indeterministic,** the actual (yes/no) outcome of a measurement is NOT uniquely determined by the input-state.

- **For the same input-state s, both answers (yes and no) can be possible**: even if we know the original state of the system.

But in this case (when we know the original state s), we can give the probability to obtain each of these outcomes.

Testable properties

A property *P is testable if there is an “experiment"*, i.e. a *yes-no measurement (with only two* possible outcomes), such that:

- If the system has property *P (BEFORE the measurement), then the result of* the measurement will surely be “YES” (with probability 1);

- If the result was “YES", then AFTER the measurement the system has property *P;*

- If the result of the measurement was “NO”, then AFTER the measurement the system does not have the property P

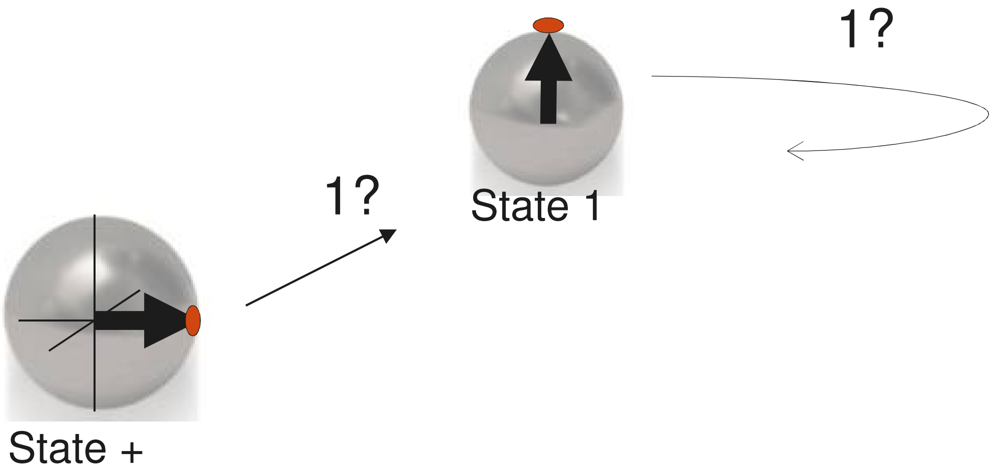
- **Testable property P is true IFF the test P? is guaranteed to be successful.**

Repeating Measurements

Even when indeterministic, quantum measurements are "consistent":

If we obtain a yes or no answer after a measurement of property P, than we will get the same answer (with probability 1) if we immediately repeat the experiment:

P? ; P? = P?

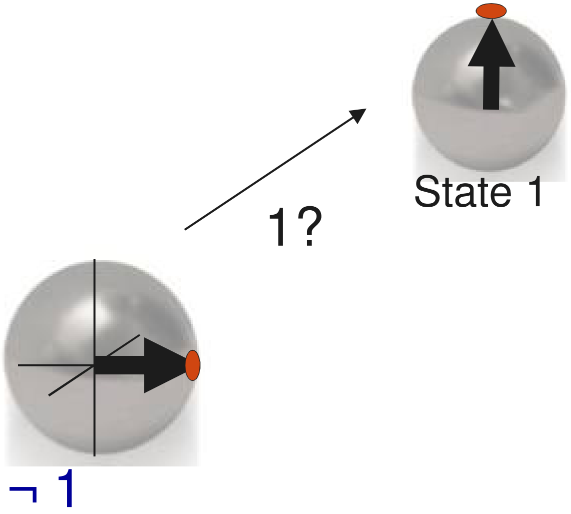


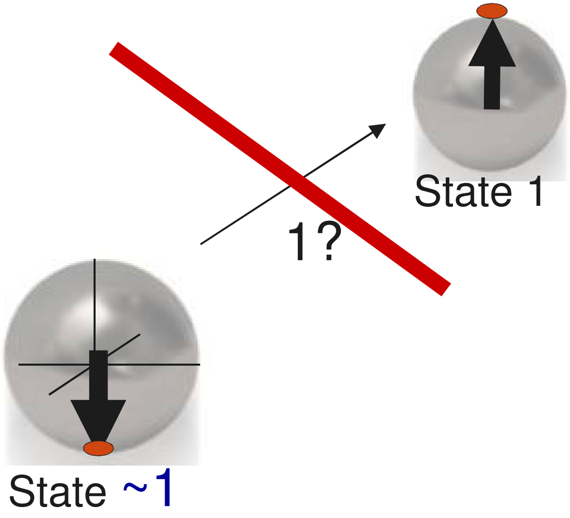
Dynamic View: Orthocomplement

- The orthocomplement (strong type of negation): ~P

It captures the fact that P is false in a certain state, and it captures the fact that EVERY test of P (in that state) will certainly fail (yield a negative result).

Only this way we can show experimentally that property P is not true in the original state.





Dynamic View: Conjunction

What is the operational meaning of the conjunction **P∧Q**

Why is the conjunction of two testable properties again a testable property?

What to do with Werner Heisenberg’s uncertainty relations?

He would say: Not possible!!?

It is impossible to make joint Measurements of position and Momentum! Or: Position and Momentum, cannot both be known to arbitrary precision.

Dynamic View: Conjunction

Back to C. Piron!

Experiments are closed under non-deterministic choice.

Given two tests P? and Q?, we can design a new experiment X as: first choosing arbitrarily either one of the two tests (P? or Q?); then performing the chosen test, and recording the answer. We see that this new experiment will surely succeed iff performing any of the two tests will surely succeed.

The testable property tested by X holds iff both P and Q hold

Disjunction and Implication

- The disjunction of testable properties is given by the Conjunction and Orthocomplement:

$$P \ \underline{v}\ Q = \sim(\sim P \wedge \sim Q)$$

- The implication:

$P \rightarrow Q$ holds iff, after successfully performing P? (on the current state of the system), property Q will surely hold (at the output state, with probability 1).

“Non-Classicality” is Dynamical

Overview:

- Classical logic is here the logic of all static properties (there is no contradiction with the principles of QM)

- The non-classical character of the logic of quantum testable properties is not a consequence of the fact that we deal with the properties of a physical system, but of the fact that they have to be “quantum testable”.

- It is the non-classical nature of quantum actions (in particular, quantum tests) that explains the strangeness of quantum behavior.

Dynamic Quantum Logic

The syntax “test-only PDL”:

$$\varphi \ ::= \ \bot \mid p \mid \varphi \wedge \varphi \mid [\,\varphi?\,]\,\varphi$$

- The weakest precondition can be understood as a dynamic measurement modality.

$[\,\varphi?\,]\,\psi$ *is the weakest precondition* ensuring that, after any successful measurement of property φ *the* system will have property ψ.

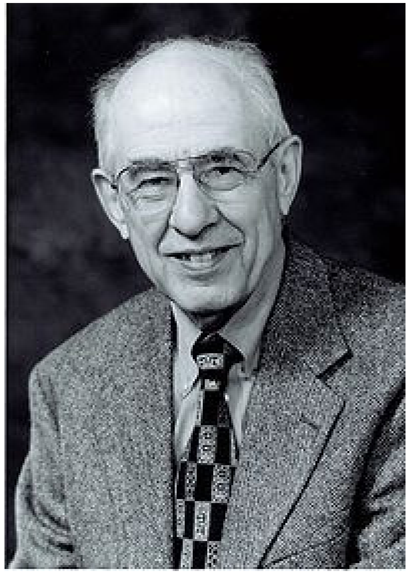
It corresponds exactly to my quantum implication (the so-called Sasaki hook).

- The orthocomplement is defined as $\sim\varphi \ := [\,\varphi?\,]\,\bot$

- **Adding unitary evolutions (basic actions), gives a nice PDL-style logic for full quantum logic (dealing with entangled systems).**

Is Logic Empirical?

- Yes, says Hilary Putnam in 1968:



"Could some of the "necessary truths" of logic ever turn out to be false *for empirical reasons*? I shall argue that the answer to this question is in the affirmative, and that logic is, in a certain sense, a natural science". (Putnam, 1968)

- Putnam's argument starts from traditional quantum logic.

- Result = endless debates about the fact that QM forces us the abandon some basic principles of classical logic.

W.E Beth, 20 years before Putnam

- At a Symposium in Scheveningen, sept. 1947

"... Nader onderzoek - van J. von Neumann (1932) e.a. - heeft doen zien, dat de aanvaarding van het complementariteits-beginsel neerkomt op een verzwakking van de logica.

... Ziehier nu een ware "crisis der zekerheden". Een ingrijpende wijziging in de logische wetten, en dan nog wel, niet krachtens een aprioristisch inzicht, maar naar aanleiding van de resultaten van empirisch onderzoek, was wel het laatste, wat men tot voor kort voor mogelijk zou hebben gehouden."

E.W. Beth, 1948 in "De Wetenschap als Cultuurfactor".

W.E Beth, 20 years before Putnam

- At a Symposium in Scheveningen, sept. 1947

- Beth argues that adopting the quantum principle of complementarity boils down to a weakening of our logic. This led in his view to a real crisis of certainties, a change of our logical laws which is due not to a priori insight but due to the results of empirical investigations.

- For Beth, a change of logic was a plausible option, as by weakening certain logical laws it becomes possible to reconcile contradictory theories.

- In my own stance I agree with the fact that quantum reality demands a logic that can express the non-classical behavior of quantum systems. But instead of working out an alternative of classical logic, our path naturally leads to a dynamic quantum logic.

Back to Putnam

What would he say about our dynamic approach to QL ?

**"To stipulate that certain sentences shall be immune from revision is *irrational* if that stipulation may lead one into serious difficulties,
such as having to postulate either mysterious disturbances by the measurement (or to say that the measurement brings what is measures into existence) or "hidden variables"....
I think it is *more likely that classical logic is wrong...*"**

(Putnam, 1968)

Counter Attack

Contrary to Putnam, I don't have any *a priori* logical intuition which shows that "tests" or other type of actions have to behave classically.

The effects of a physical action are a matter of "experiment", not of a priori reasoning.

PART 3: Applications

-Teleportation and Quantum Key Distribution

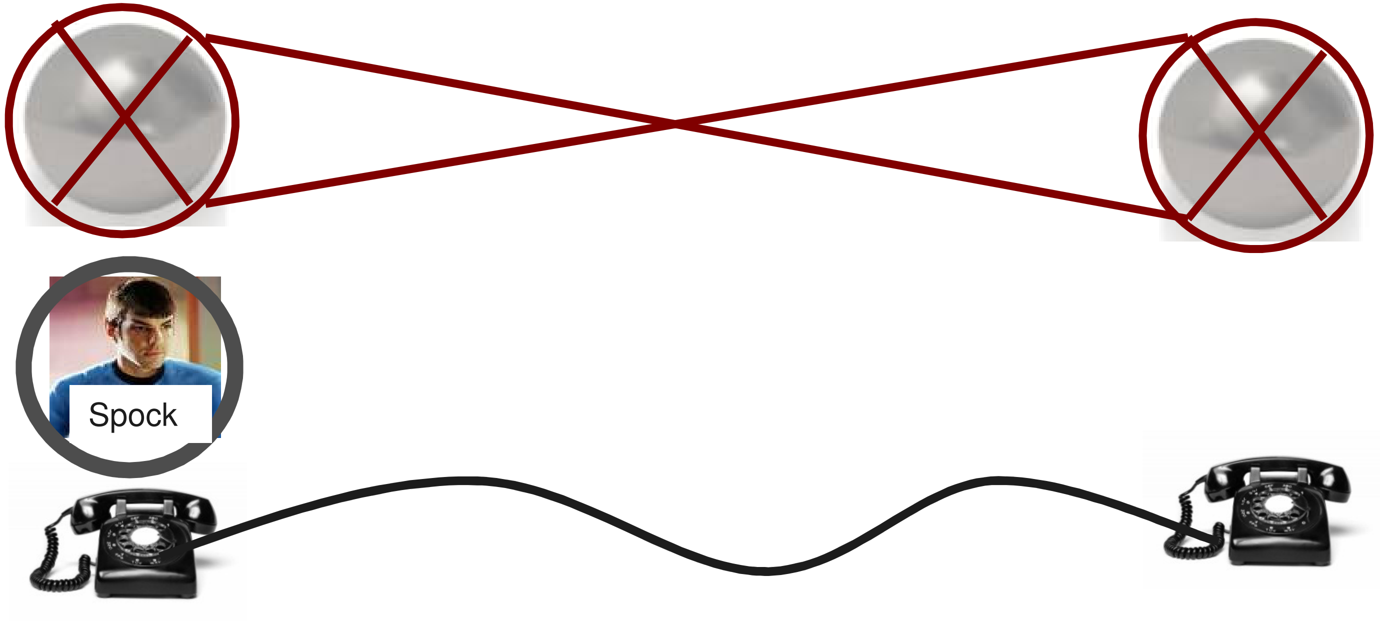
Protocol : Quantum teleportation

- Technique to teleport the state of a quantum system


Alice

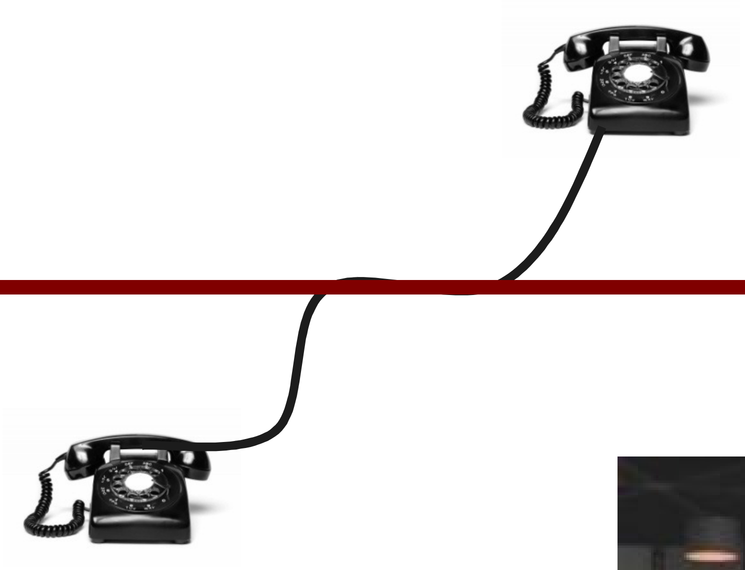

Bob

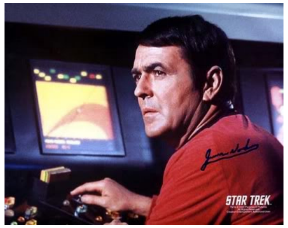
- Alice will teleport her qubit (“Spock”) to Bob.
- Use entangled qubits and classical communication.



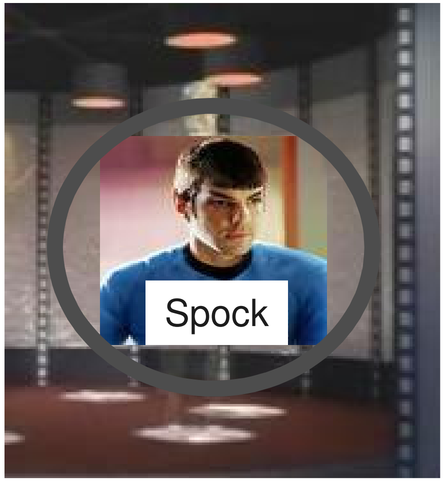
- Alice sends her qubits to logical gates (CNOT, Hadamard)

- she measures the result and calls Bob





- Bob performs a correction, depending on Alice’s measurement result



Quantum communication

- Quantum Key Distribution Protocol (Bennett & Brassard 84)

  - Principle: Observation causes perturbation
  - "eavesdropping" and "wiretapping" can be detected with a very high chance.

  - "The basic idea is to exploit the quantum mechanical principle that observation in general disturbs the system being observed. Thus, if there is an eavesdropper listening in as Alice and Bob attempt to transmit their key, the presence of the eavesdropper will be visible as a disturbance of the communications channel Alice and Bob are using to establish the key." (Nielsen & Chuang)

Quantum communication

- Commercially available:

id Quantique

A Quantum Leap For Cryptography

➢ Used in 2007 in Geneve to transmit ballots during the national elections.

Logic used to verify protocols:

- Does the program (protocol) what it is supposed to do?
- Logical proof of the correctness or safety of the protocol.
- How? Via “theorem proving”:
  - Logical encoding van the desired protocol en the desired program specification in dynamic quantum logic: $\varphi \rightarrow [\,\pi\,]\,\psi$

Thank You!

http://sonja.tiddlyspot.com