# Choosing $k$ from $m$ 

Bezalel Peleg, Hans Peters

Amsterdam, 19-03-2015

## How it started

"One year, the department was asked by the Dean to suggest two people for slots that were opening up in the Faculty of Natural Sciences and Mathematics. Four serious mathematicians were candidates; [...after the committee selection it turned out that...] not only [was] most of the department opposed to last night's decision, but there [was] even a specific pair that most of the department prefers to the one chosen [...]"
R.J. Aumann (2012) My scientific first-born. Special issue of International Journal of Game Theory in honor of Bezalel Peleg.

## The central question

- There are $m$ candidates, from which a committee of size $k$ has to be chosen: $1 \leq k \leq m-1$.
- There are $n$ voters with linear preferences on the set of candidates.
- Is there a voting method such that no coalition of voters, by voting strategically, can guarantee a committee that all voters in the coalition prefer to the (or any) committee chosen by truthful voting?

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

We apply Borda with weights $3,2,1,0$.

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

We apply Borda with weights $3,2,1,0$. In left profile: $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$.

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

We apply Borda with weights $3,2,1,0$.
In left profile: $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$. Say $(b, a)$ is chosen (according to some tie-breaking rule).

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

We apply Borda with weights $3,2,1,0$.
In left profile: $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$. Say $(b, a)$ is chosen (according to some tie-breaking rule).
In right profile: $\{(b, c)\}$.
(Second alternative: "chairman")

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

We apply Borda with weights $3,2,1,0$.
In left profile: $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$. Say $(b, a)$ is chosen (according to some tie-breaking rule).
In right profile: $\{(b, c)\}$.
(Second alternative: "chairman")
Lexicographic preferences over sets:

- worst first
- chairman first

In both cases, coalition $\{2,3\}$ "manipulates".

An example: $n=3, m=4, k=2$

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ | $a$ | $c$ | $c$ |
| $b$ | $c$ | $a$ | $b$ | $b$ | $b$ |
| $c$ | $a$ | $b$ | $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | $d$ | $d$ | $d$ |

We apply Borda with weights $3,2,1,0$.
In left profile: $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$. Say $(b, a)$ is chosen (according to some tie-breaking rule).
In right profile: $\{(b, c)\}$.
(Second alternative: "chairman")
Lexicographic preferences over sets:

- worst first
- chairman first

In both cases, coalition $\{2,3\}$ "manipulates".
Similarly for other choices in left profile.

## The same example, now with FEP

Each alternative gets weight one. We eliminate alternatives and preferences at the same time, from bottom up. For instance:

| $R^{1}$ | $R^{2}$ | $R^{3}$ |  | $R^{2}$ | $R^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a$ | $b$ | $c$ |  |  |  |
| $b$ | $c$ | $a$ | El. $d, R^{1} \rightarrow$ | $b$ | $c$ |
| $c$ | $a$ | El. $a, R^{2} \rightarrow(b, c)$ |  |  |  |
| $c$ | $a$ | $b$ |  | $a$ | $b$ |

This way we get $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$, say $(b, a)$ is chosen.

## The same example, now with FEP

Each alternative gets weight one. We eliminate alternatives and preferences at the same time, from bottom up. For instance:

| $R^{1}$ | $R^{2}$ | $R^{3}$ |
| :---: | :---: | :---: |
| $a$ | $b$ | $c$ |
| $b$ | $c$ | $a$ |
| $c$ | $a$ | $b$ |
| $d$ | $d$ | $d$ |

$R^{2} \quad R^{3}$

This way we get $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$, say $(b, a)$ is chosen. For the profile

| $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: |
| $a$ | $c$ | $c$ |
| $b$ | $b$ | $b$ |
| $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ | we get $\{(a, b),(c, b),(b, c)\}$.

## The same example, now with FEP

Each alternative gets weight one. We eliminate alternatives and preferences at the same time, from bottom up. For instance:

| $R^{1}$ | $R^{2}$ | $R^{3}$ |
| :---: | :---: | :---: |
| $a$ | $b$ | $c$ |
| $b$ | $c$ | $a$ |
| $c$ | $a$ | $b$ |
| $d$ | $d$ | $d$ |

$R^{2} \quad R^{3}$

This way we get $\{(a, b),(b, a),(a, c),(c, a),(b, c),(c, b)\}$, say $(b, a)$ is chosen.For the profile

| $R^{1}$ | $Q^{2}$ | $Q^{3}$ |
| :---: | :---: | :---: |
| $a$ | $c$ | $c$ |
| $b$ | $b$ | $b$ |
| $c$ | $a$ | $a$ |
| $d$ | $d$ | $d$ |$\quad$ we get $\{(a, b),(c, b),(b, c)\}$.

These are not all preferred to $(b, a)$ for voters 2 and 3: these voters cannot guarantee something better.

## Outlook of the paper and presentation

- We focus on FEP, Feasible Elimination Procedures.


## Outlook of the paper and presentation

- We focus on FEP, Feasible Elimination Procedures.
- Background: FEP were introduced as an "escape" from the Gibbard-Satterthwaite result (Peleg, 1978).


## Outlook of the paper and presentation

- We focus on FEP, Feasible Elimination Procedures.
- Background: FEP were introduced as an "escape" from the Gibbard-Satterthwaite result (Peleg, 1978).
- We show how FEP can be used to choose $k$ from $m$.


## Outlook of the paper and presentation

- We focus on FEP, Feasible Elimination Procedures.
- Background: FEP were introduced as an "escape" from the Gibbard-Satterthwaite result (Peleg, 1978).
- We show how FEP can be used to choose $k$ from $m$.
- We consider computation: equivalent to finding maximal matchings in bipartite graphs.


## Outlook of the paper and presentation

- We focus on FEP, Feasible Elimination Procedures.
- Background: FEP were introduced as an "escape" from the Gibbard-Satterthwaite result (Peleg, 1978).
- We show how FEP can be used to choose $k$ from $m$.
- We consider computation: equivalent to finding maximal matchings in bipartite graphs.
- We have an axiomatic characterization for the case $k=1$ (not in this presentation).


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.
- $N$ is the finite set of voters, $|N|=n \geq 2$.


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.
- $N$ is the finite set of voters, $|N|=n \geq 2$.
- $L$ is the set of preferences $(=$ linear orderings) on $A$.


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.
- $N$ is the finite set of voters, $|N|=n \geq 2$.
- $L$ is the set of preferences ( $=$ linear orderings) on $A$.
- A social choice function is a map $F: L^{N} \rightarrow A$.


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.
- $N$ is the finite set of voters, $|N|=n \geq 2$.
- $L$ is the set of preferences ( $=$ linear orderings) on $A$.
- A social choice function is a map $F: L^{N} \rightarrow A$.
- A pair $\left(F, R^{N}\right)$ with $R^{N} \in L^{N}$ is a(n ordinal) voting game.


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.
- $N$ is the finite set of voters, $|N|=n \geq 2$.
- $L$ is the set of preferences ( $=$ linear orderings) on $A$.
- A social choice function is a map $F: L^{N} \rightarrow A$.
- A pair $\left(F, R^{N}\right)$ with $R^{N} \in L^{N}$ is a(n ordinal) voting game.
- $F$ is non-manipulable (or strategy-proof) if $R^{N}$ is a Nash equilibrium in $\left(F, R^{N}\right)$ for every $R^{N} \in L^{N}$.


## Basic model and preliminaries

- $A$ is the finite set of alternatives, $|A|=m \geq 2$.
- $N$ is the finite set of voters, $|N|=n \geq 2$.
- $L$ is the set of preferences ( $=$ linear orderings) on $A$.
- A social choice function is a map $F: L^{N} \rightarrow A$.
- A pair $\left(F, R^{N}\right)$ with $R^{N} \in L^{N}$ is a(n ordinal) voting game.
- $F$ is non-manipulable (or strategy-proof) if $R^{N}$ is a Nash equilibrium in $\left(F, R^{N}\right)$ for every $R^{N} \in L^{N}$.
- THEOREM (Gibbard, 1973; Satterthwaite, 1975). Let $F$ be non-manipulable with at least three alternatives in its range. Then $F$ is dictatorial.
- Social choice function $F$ is exactly and strongly consistent (ESC) if for every $R^{N} \in L^{N}$ there is a strong Nash equilibrium $Q^{N}$ of $\left(F, R^{N}\right)$ such that $F\left(Q^{N}\right)=F\left(R^{N}\right)$. (Peleg, 1978)
- Social choice function $F$ is exactly and strongly consistent (ESC) if for every $R^{N} \in L^{N}$ there is a strong Nash equilibrium $Q^{N}$ of $\left(F, R^{N}\right)$ such that $F\left(Q^{N}\right)=F\left(R^{N}\right)$. (Peleg, 1978)
- In other words, for an ESC social choice function there is for every profile of true preferences a strong Nash equilibrium of the voting game that results in the sincere (truthful) outcome.
- In order to obtain ESC social choice functions, feasible elimination procedures play a crucial role.


## Feasible elimination procedures

## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.
- Eliminate $x$ everywhere from the profile.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.
- Eliminate $x$ everywhere from the profile.
- Repeat the procedure for the remaining profile with $\beta(x)$ voters less and without $x$.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.
- Eliminate $x$ everywhere from the profile.
- Repeat the procedure for the remaining profile with $\beta(x)$ voters less and without $x$.
- Continue doing this until all but one alternatives have been eliminated.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.
- Eliminate $x$ everywhere from the profile.
- Repeat the procedure for the remaining profile with $\beta(x)$ voters less and without $x$.
- Continue doing this until all but one alternatives have been eliminated.
- The remaining alternative is the outcome of the procedure.


## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.
- Eliminate $x$ everywhere from the profile.
- Repeat the procedure for the remaining profile with $\beta(x)$ voters less and without $x$.
- Continue doing this until all but one alternatives have been eliminated.
- The remaining alternative is the outcome of the procedure.

The resulting alternative(s) is (are) called $R^{N}$-maximal.

## Feasible elimination procedures

- Let $R^{N}$ be a profile of preferences.
- Assign weights $\beta(x) \in \mathbb{N}$ to the alternatives $x \in A$ such that $\sum_{x \in A} \beta(x)=n+1$.
- Find an alternative $x$ that occurs at bottom for at least $\beta(x)$ many voters.
- Pick exactly $\beta(x)$ voters who have $x$ at bottom and eliminate their preferences from the profile.
- Eliminate $x$ everywhere from the profile.
- Repeat the procedure for the remaining profile with $\beta(x)$ voters less and without $x$.
- Continue doing this until all but one alternatives have been eliminated.
- The remaining alternative is the outcome of the procedure.

The resulting alternative(s) is (are) called $R^{N}$-maximal.
Note that all this depends on the exogenously chosen weights.

## An example

## An example

$$
A=\{a, b, c\}, N=\{1, \ldots, 5\}, \beta(a)=\beta(b)=\beta(c)=2 .
$$

## An example

$$
A=\{a, b, c\}, N=\{1, \ldots, 5\}, \beta(a)=\beta(b)=\beta(c)=2 .
$$

$$
\begin{array}{ccccc}
R^{1} & R^{2} & R^{3} & R^{4} & R^{5} \\
b & c & a & c & a \\
c & b & b & a & c \\
a & a & c & b & b
\end{array}
$$

## An example

$$
\left.\begin{array}{c}
A=\{a, b, c\}, N=\{1, \ldots, 5\}, \beta(a)=\beta(b)=\beta(c)=2 . \\
R^{1} \\
b \\
R^{2} \\
R^{3}
\end{array}\right) R^{4} \quad R^{5} .
$$

There are two FEPs:

## An example

$A=\{a, b, c\}, N=\{1, \ldots, 5\}, \beta(a)=\beta(b)=\beta(c)=2$.

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{4}$ | $R^{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| $b$ | $c$ | $a$ | $c$ | $a$ |
| $c$ | $b$ | $b$ | $a$ | $c$ |
| $a$ | $a$ | $c$ | $b$ | $b$ |

There are two FEPs:
$(a,\{1,2\} ; b,\{4,5\} ; c)$

## An example

$A=\{a, b, c\}, N=\{1, \ldots, 5\}, \beta(a)=\beta(b)=\beta(c)=2$.

| $R^{1}$ | $R^{2}$ | $R^{3}$ | $R^{4}$ | $R^{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| $b$ | $c$ | $a$ | $c$ | $a$ |
| $c$ | $b$ | $b$ | $a$ | $c$ |
| $a$ | $a$ | $c$ | $b$ | $b$ |

There are two FEPs:
$(a,\{1,2\} ; b,\{4,5\} ; c)$
$(b,\{4,5\} ; a,\{1,2\} ; c)$.

## An example

$A=\{a, b, c\}, N=\{1, \ldots, 5\}, \beta(a)=\beta(b)=\beta(c)=2$.

$$
\begin{array}{ccccc}
R^{1} & R^{2} & R^{3} & R^{4} & R^{5} \\
b & c & a & c & a \\
c & b & b & a & c \\
a & a & c & b & b
\end{array}
$$

There are two FEPs:
$(a,\{1,2\} ; b,\{4,5\} ; c)$
(b, \{4, 5\}; a, $\{1,2\} ; c)$.
Hence $c$ is the only $R^{N}$-maximal alternative.

Relevance of FEP

## Relevance of FEP

- Voter $i \in N$ is a vetoer for social choice function $F$ is there is an alternative $x \in A$ and a preference for voter $i$ such that by reporting this preference $i$ can guarantee that $x$ is not chosen.


## Relevance of FEP

- Voter $i \in N$ is a vetoer for social choice function $F$ is there is an alternative $x \in A$ and a preference for voter $i$ such that by reporting this preference $i$ can guarantee that $x$ is not chosen.
- THEOREM (Peleg and Peters 2010) Let $F$ be anonymous without vetoers. Then $F$ is ESC if and only if there is a weight function $\beta$ such that $F$ assigns an $R^{N}$-maximal alternative to every profile $R^{N}$.


## Relevance of FEP

- Voter $i \in N$ is a vetoer for social choice function $F$ is there is an alternative $x \in A$ and a preference for voter $i$ such that by reporting this preference $i$ can guarantee that $x$ is not chosen.
- THEOREM (Peleg and Peters 2010) Let $F$ be anonymous without vetoers. Then $F$ is ESC if and only if there is a weight function $\beta$ such that $F$ assigns an $R^{N}$-maximal alternative to every profile $R^{N}$.
- Remark: note that $\beta(x) \geq 2$ for any $\beta$ in this Theorem and any $x \in A$.


## Relevance of FEP

- Voter $i \in N$ is a vetoer for social choice function $F$ is there is an alternative $x \in A$ and a preference for voter $i$ such that by reporting this preference $i$ can guarantee that $x$ is not chosen.
- THEOREM (Peleg and Peters 2010) Let $F$ be anonymous without vetoers. Then $F$ is ESC if and only if there is a weight function $\beta$ such that $F$ assigns an $R^{N}$-maximal alternative to every profile $R^{N}$.
- Remark: note that $\beta(x) \geq 2$ for any $\beta$ in this Theorem and any $x \in A$.
- Goes back to results by Peleg (1978), Holzman (1986), and others.


## Choosing $k$ from $m$

The idea is to use FEPs to choose committees of $k$ candidates from in total $m$ candidates. For instance, for $k=2$ :

$$
\begin{array}{ccccc}
R^{1} & R^{2} & R^{3} & R^{4} & R^{5} \\
b & c & a & c & a \\
c & b & b & a & c \\
a & a & c & b & b
\end{array}
$$

## Choosing $k$ from $m$

The idea is to use FEPs to choose committees of $k$ candidates from in total $m$ candidates. For instance, for $k=2$ :

$$
\begin{array}{ccccc}
R^{1} & R^{2} & R^{3} & R^{4} & R^{5} \\
b & c & a & c & a \\
c & b & b & a & c \\
a & a & c & b & b
\end{array}
$$

$(a,\{1,2\} ; b,\{4,5\} ; c)$ results in $(b, c)$.

## Choosing $k$ from $m$

The idea is to use FEPs to choose committees of $k$ candidates from in total $m$ candidates. For instance, for $k=2$ :

$$
\begin{array}{ccccc}
R^{1} & R^{2} & R^{3} & R^{4} & R^{5} \\
b & c & a & c & a \\
c & b & b & a & c \\
a & a & c & b & b
\end{array}
$$

$(a,\{1,2\} ; b,\{4,5\} ; c)$ results in $(b, c)$.
$(b,\{4,5\} ; a,\{1,2\} ; c)$ results in $(a, c)$.

- In the paper we consider mainly two preference extensions.
- Lexicographic worst Lexicographic comparison starting from worst alternative.

Example: $m=5, k=3$. Preference $a b c d e$. Then $(d, c, a)$ is preferred over ( $b, a, e$ ) and over ( $b, d, c$ ). (Order of alternatives is irrelevant.)

- In the paper we consider mainly two preference extensions.
- Lexicographic worst Lexicographic comparison starting from worst alternative.

Example: $m=5, k=3$. Preference $a b c d e$. Then $(d, c, a)$ is preferred over ( $b, a, e$ ) and over ( $b, d, c$ ). (Order of alternatives is irrelevant.)

- Lexicographic from top Lexicographic comparison starting from the right.

Example: $m=5, k=3$. Preference $a b c d e$. Then $(e, a, b)$ is preferred over $(b, a, c)$ and over $(e, c, b)$, but not over $(e, d, a)$. (Order of alternatives can matter.)

## Main result

## Main result

## Theorem

Suppose we choose a committee of $k$ members from a set of $m$ alternatives according to a feasible elimination procedure $(1 \leq k \leq m-1)$. Assume the lexicographic worst or lexicographic from top preference extension. Then there is no coalition who can guarantee (by reporting some preference profile) a committee that is strictly preferred by all members of the coalition to the sincere one (any committee selected from the set of sincere committees).

- This result does not hold for other methods, e.g., scoring rules like Borda's, STV, etc.
- This result does not hold for other methods, e.g., scoring rules like Borda's, STV, etc.
- The result does not hold for (e.g.) the lexicographic best preference extension.
- This result does not hold for other methods, e.g., scoring rules like Borda's, STV, etc.
- The result does not hold for (e.g.) the lexicographic best preference extension.
- The procedure cannot always be made neutral (all weights equal). But if the number of voters is relatively large then this does not matter too much, e.g., $m=10, n=1000$ : take nine weights equal to 100 and one weight equal to 101 .


## Computation

## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee $\left(a_{1}, \ldots, a_{k}\right)$.


## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee $\left(a_{1}, \ldots, a_{k}\right)$.
- To check: can $\left(a_{1}, \ldots, a_{k}\right)$ result from FEP?


## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee ( $a_{1}, \ldots, a_{k}$ ).
- To check: can $\left(a_{1}, \ldots, a_{k}\right)$ result from FEP?
- Make bipartite graph with $n$ left hand nodes (voters) and $\beta(x)$ right hand nodes per alternative $x \neq a_{k}$.


## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee $\left(a_{1}, \ldots, a_{k}\right)$.
- To check: can $\left(a_{1}, \ldots, a_{k}\right)$ result from FEP?
- Make bipartite graph with $n$ left hand nodes (voters) and $\beta(x)$ right hand nodes per alternative $x \neq a_{k}$.
- For $x$ not in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members to $x$.


## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee $\left(a_{1}, \ldots, a_{k}\right)$.
- To check: can $\left(a_{1}, \ldots, a_{k}\right)$ result from FEP?
- Make bipartite graph with $n$ left hand nodes (voters) and $\beta(x)$ right hand nodes per alternative $x \neq a_{k}$.
- For $x$ not in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members to $x$.
- For $x=a_{\ell}$ in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members $a_{j}, j>\ell$, to $x$.


## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee $\left(a_{1}, \ldots, a_{k}\right)$.
- To check: can $\left(a_{1}, \ldots, a_{k}\right)$ result from FEP?
- Make bipartite graph with $n$ left hand nodes (voters) and $\beta(x)$ right hand nodes per alternative $x \neq a_{k}$.
- For $x$ not in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members to $x$.
- For $x=a_{\ell}$ in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members $a_{j}, j>\ell$, to $x$.
- LEMMA: $\left(a_{1}, \ldots, a_{k}\right)$ can result from FEP if and only if this graph has a maximal matching.


## Computation

- Given are profile $R^{N}$, weights $\beta(x)$, committee ( $a_{1}, \ldots, a_{k}$ ).
- To check: can $\left(a_{1}, \ldots, a_{k}\right)$ result from FEP?
- Make bipartite graph with $n$ left hand nodes (voters) and $\beta(x)$ right hand nodes per alternative $x \neq a_{k}$.
- For $x$ not in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members to $x$.
- For $x=a_{\ell}$ in the committee, draw edges between $i \in N$ on the left and all $\beta(x)$ on the right if $i$ prefers all committee members $a_{j}, j>\ell$, to $x$.
- LEMMA: $\left(a_{1}, \ldots, a_{k}\right)$ can result from FEP if and only if this graph has a maximal matching.
- This can be checked in polynomial time (Hopcroft and Karp, 1973). Repeating this procedure $m(m-1) \cdots(m-k+1)$ times is still polynomial.


## Concluding remarks

## Concluding remarks

- There are many other, non-strategic approaches to choosing committees in the literature. For instance, approaches based on pairwise majority relations (Condorcet winners/losers). See the recent thesis of Eric Kamwa (Essais sur les modes de scrutins et la sélection des comités, Caen, 2014).


## Concluding remarks

- There are many other, non-strategic approaches to choosing committees in the literature. For instance, approaches based on pairwise majority relations (Condorcet winners/losers). See the recent thesis of Eric Kamwa (Essais sur les modes de scrutins et la sélection des comités, Caen, 2014).
- In the paper we argue that methods based on pairwise majority do not have the core property and, hence, are sensitive to manipulation by coalitions.


## Concluding remarks

- There are many other, non-strategic approaches to choosing committees in the literature. For instance, approaches based on pairwise majority relations (Condorcet winners/losers). See the recent thesis of Eric Kamwa (Essais sur les modes de scrutins et la sélection des comités, Caen, 2014).
- In the paper we argue that methods based on pairwise majority do not have the core property and, hence, are sensitive to manipulation by coalitions.
- In the paper most of our results are framed in terms of cores of effectivity functions. A method has the core property of it assigns committees belonging to the core of the associated effectivity function.


## Concluding remarks

- There are many other, non-strategic approaches to choosing committees in the literature. For instance, approaches based on pairwise majority relations (Condorcet winners/losers). See the recent thesis of Eric Kamwa (Essais sur les modes de scrutins et la sélection des comités, Caen, 2014).
- In the paper we argue that methods based on pairwise majority do not have the core property and, hence, are sensitive to manipulation by coalitions.
- In the paper most of our results are framed in terms of cores of effectivity functions. A method has the core property of it assigns committees belonging to the core of the associated effectivity function.
- The (an) axiomatization for $k>1$ is still open.


## Concluding remarks

- There are many other, non-strategic approaches to choosing committees in the literature. For instance, approaches based on pairwise majority relations (Condorcet winners/losers). See the recent thesis of Eric Kamwa (Essais sur les modes de scrutins et la sélection des comités, Caen, 2014).
- In the paper we argue that methods based on pairwise majority do not have the core property and, hence, are sensitive to manipulation by coalitions.
- In the paper most of our results are framed in terms of cores of effectivity functions. A method has the core property of it assigns committees belonging to the core of the associated effectivity function.
- The (an) axiomatization for $k>1$ is still open.
- There are (open) issues concerning neutrality, other preference extensions.


## THE END

