Reason-Based Preferences and Preference Aggregation

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3rd ILLC Workshop on Collective Decision Making, June 2019

Overview

- A reason-based model of preferences.
- Weighted Description Logics and concept combinations.
- Individual preferences and combinations of reasons.
- Setting the problem of reason-based preference aggregation.

A reason-based model of preferences

(Dietrich and List, 2013)

- Given a set of alternatives X (policies, candidates of an election, goods to be allocated). Individuals express their preferences (indifference) $\succeq$ ($\sim$) on X.
- A *reason* is a *property* of elements of X, that is a subset $R \subseteq X$ (extensional view of properties)
- A set of *motivating* reasons M is set of reasons that motivates a preference ordering $\succeq_M$ (or indifference $\sim_M$) on X.
- The dependency of preferences on reasons is captured by two axioms.

A reason-based model of preferences

- Axiom 1: if $\{R \in M \ s.t. R(x)\} = \{R \in M \ s.t. R(y)\}$ then $x \sim_M y$.
- Axiom 2: for any x, y in X and any M, M' in $\mathcal{P}(X)$ with $M \subseteq M'$, if no R in M' is true of x and y, then $x \succeq_M y$ iff $x \succeq_{M'} y$.

Those axioms hold, iff it is possible to associate to each preference relation a weight on the relevance of the sets of reasons:

Theorem 1

For $M \in \mathcal{P}(X)$, $x \succeq_M y$ satisfy Axiom 1 and Axiom 2 iff there exists a weighing relation $\geq$ such that, for all x, $y \in X$:

$$x \succeq_M y \text{ iff } \{R \in M \ s.t. R(x)\} \geq \{R \in M \ s.t. R(y)\}$$

A plan

- We introduce *Description Logics* (DLs) as a suitable framework to reason about combinations of properties (aka concepts).
- We extend DLs by admitting weighted formulas and complex concept constructors.
- We discuss ho to model the relationship between individual preferences and combinations of concepts.
  - Dependency of preferences on the applicable concepts,
  - Contextual dependency of concepts satisfaction,
  - Expressive weighing of concept combinations.

Description logics ($\mathcal{ALC}$).

- The language of $\mathcal{ALC}$ is based on an alphabet consisting of *atomic concepts*, *role names*, and *object names*.
- The set of *concept descriptions* is generated as follows (where A represents atomic concepts and R role names):

$$C \quad ::= \quad A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C$$

- A *TBox* is a finite set of formulas of the form $A \sqsubseteq C$ and $A \equiv C$ (where A is an atomic concept and C a concept description).
- An *ABox* is a finite set of formulas of the form $A(a)$ and $R(a, b)$.

Description logics ($\mathcal{ALC}$). Semantics

- The semantics of $\mathcal{ALC}$ is given by *interpretations* $I = (\Delta^I, \cdot^I)$.
- $\cdot^I$ maps each object name to an element of Δ^I, each atomic concept to a subset of the domain, and each role name to a binary relation on the domain.
- $\cdot^I$ extends to complex concepts by:

$$(\neg C)^I = \Delta^I \setminus C^I$$
$$(C \sqcap D)^I = C^I \cap D^I$$
$$(\forall R.C)^I = \{d \in \Delta^I \mid \forall e,\ (d,e) \in R^I \Rightarrow e \in C^I\}$$
$$(\exists R.C)^I = \{d \in \Delta^I \mid \exists e,\ (d,e) \in R^I \;\&\; e \in C^I\}$$

- $C(a)$ is true in I iff $a^I \in C^I$. $R(a,b)$ is true in I iff $(a^I, b^I) \in R^I$.
- $C \sqsubseteq D$ is true in I iff $C^I \subseteq D^I$.
- A set of (TBox and ABox) formulas is *satisfiable* if there exists an interpretation in which they are all true.

Example

- TaxHighIncomes(b)
- RaiseWelfare(a)
- RaiseWages(b)
- TaxHighIncomes(a)
- RecudeTaxation(c)

- ReduceTaxation $\sqsubset \neg$TaxHighIncomes
- RaiseWelfare $\sqsubseteq$ LeftPolicy
- RaiseWages $\sqsubseteq$ LeftPolicy
- RaiseWelfare $\sqsubseteq \neg$RaiseWages
- TaxHighIncomes $\sqsubseteq$ LeftPolicy
- LeftPolicy $\sqsubseteq$ RaiseWages $\sqcup$ RaiseWelfare $\sqcup$ TaxHighIncomes
- LeftPolicy $\sqsubseteq \exists$hasConsequence.ReduceInequality

Weighted concepts combinations

- We introduce *weighted concept descriptions* to model operators $\mathbb{W}$ (spoken "tooth") that:
    - *i* take a list of concept descriptions,
    - *ii* associate a vector of weights to them,
    - *iii* return a complex concept that applies to those instances that satisfy a certain combination of concepts, reaching a certain threshold.
- The new logic is denoted by $\mathcal{ALC}_{\mathbb{W}^{\mathbb{R}}}$, where weights and thresholds range over real numbers $r \in \mathbb{R}$

The set of $\mathcal{ALC}_{\mathbb{W}}$ concepts is then described by the grammar:

$$C \quad ::= \quad A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C \mid \mathbb{W}^{t}_{\vec{w}}(C_1, \ldots, C_m)$$

Weighted concepts combinations. Semantics

Given $\mathtt{C} = (C_1, w_1), \ldots, (C_m, w_m)$,

- Let $I = (\Delta^I, \cdot^I)$ be an interpretation of $\mathcal{ALC}$. We define the *value* of a $d \in \Delta^I$ under $\mathtt{C}$ by setting:

$$v_{\mathtt{C}}^{I}(d) = \sum_{i \in \{1,\ldots,m\}} \{w_i \mid d \in C_i^I\} \quad (1)$$

- Let a be an object name of $\mathcal{ALC}$ and $\mathcal{K}$ an $\mathcal{ALC}$ knowledge base. We set the *value* of a in $\mathcal{K}$ by:

$$v_{\mathtt{C}}^{\mathcal{K}}(a) := \sum_{i \in \{1,\ldots,m\}} \{w_i \mid \mathcal{K} \models C_i(a)\} \quad (2)$$

  I.e., $v_{\mathtt{C}}^{\mathcal{K}}(a)$ gives the accumulated weight of those C_i that are entailed by $\mathcal{K}$ to satisfy a.

Weighted concept combinations. Operators

We can introduce two concept constructions:

- A $\nabla\!\!\!\!\nabla^{t}_{\vec{w}}(C_1, \ldots, C_m)$ which applies to the elements that reach a certain threshold t:

$$(\nabla\!\!\!\!\nabla^{t}_{\vec{w}}(C_1, \ldots, C_m))^I = \{d \in \Delta^I \mid v^I_{\mathtt{C}}(d) \geq t\} \tag{3}$$

- A $\nabla\!\!\!\!\nabla^{max}_{\vec{w}}(C_1, \ldots, C_m)$ which applies to the instances that maximise the possible score:

$$\left(\nabla\!\!\!\!\nabla^{max}((C_1, w_1), \ldots (C_m, w_m))\right)^I = \{d \in \Delta \mid v^I_{\mathtt{C}}(d) \geq v^I_{\mathtt{C}}(d') \text{ for all } d' \in \Delta\} \tag{4}$$

Example

Suppose that a knowledge base $\mathcal{K}$ is given:

$$\mathcal{K} = \{\mathtt{RaiseWages}(\mathtt{a}), \mathtt{ReduceTaxation}(b), \mathtt{RaiseWages} \sqsubseteq \mathtt{LeftPolicy},$$
$$\mathtt{LeftPolicy} \sqsubseteq \exists \mathtt{hasConsequence.ReduceInequality},$$
$$\mathtt{ReduceTaxation} \sqcap \exists \mathtt{hasConsequence.ReduceInequality} \sqsubseteq \bot\}$$

- Suppose $\mathtt{C}$ defined by means of the $\mathbb{W}^t$ operator:

  $$\mathtt{C} = \mathbb{W}^t((\mathtt{RaiseWages}, 2), (\exists \mathtt{hasConsequence.ReduceInequality}, 3), (\mathtt{ReduceTaxation}, 4))$$

  We have that $v_{\mathtt{C}}^{\mathcal{K}}(a) = 2 + 3$ and $v_{\mathtt{C}}^{\mathcal{K}}(b) = 4$. If $t = 4$, both $\mathtt{C}(a)$ and $\mathtt{C}(b)$.

- Suppose $\mathtt{C}$ defined by means of the $\mathbb{W}^{max}$ operator:

  $$\mathtt{C} = \mathbb{W}^{max}((\mathtt{RaiseWages}, 2), (\exists \mathtt{hasConsequence.ReduceInequality}, 3), (\mathtt{ReduceTaxation}, 4))$$

  Here, we only get that $\mathtt{C}(a)$, since for no instances we can satisfy the three concepts in $\mathtt{C}$.

Properties of the $\mathbb{W}$ operators

Firstly, we note that, for every possible choice of weights and thresholds, the operators are well-defined, the $\mathbb{W}$s of equivalent concepts return equivalent concepts:

$$C_i^I = D_i^I \implies (\mathbb{W}^t_{\vec{w}}(C_1, \ldots, C_i, \ldots, C_m))^I = (\mathbb{W}^t_{\vec{w}}(C_1, \ldots, D_i, \ldots, C_m))^I$$

$$\textit{and}$$

$$(\mathbb{W}^{max}_{\vec{w}}(C_1, \ldots, C_i, \ldots, C_m))^I = (\mathbb{W}^{max}_{\vec{w}}(C_1, \ldots, D_i, \ldots, C_m))^I$$

A number of other properties depend on the choice of the set of weights, e.g, for $w_i \in \mathbb{R}^+_0$ we have:

$$C_i^I \subseteq D_i^I \implies (\mathbb{W}^t_{\vec{w}}(C_1, \ldots, C_i, \ldots, C_m))^I \subseteq (\mathbb{W}^t_{\vec{w}}(C_1, \ldots, D_i, \ldots, C_m))^I \quad (5)$$

Expressivity

- $\nabla\!\!\!\!\nabla^t$ does not increase the expressive power of $\mathcal{ALC}$.
- By contrast, $\nabla\!\!\!\!\nabla^{max}$ does, as it allows for defining the universal role (U) quantification $\forall U.C$, which is outside $\mathcal{ALC}$:

$$\forall U.C \equiv \left(\nabla\!\!\!\!\nabla^{max}_{(-1)}(C)\right) \sqcap C \tag{6}$$

- By means of $\nabla\!\!\!\!\nabla^t$ we can present (possibly) succinct definition of :

$$\text{A majority of } \{C_1, \ldots, C_m\} \text{ applies:} \nabla\!\!\!\!\nabla^m((C_1, 2), \ldots, (C_m, 2)) \tag{7}$$

$$\nabla\!\!\!\!\nabla^{\leq t}((C_1, w_1), \ldots (C_m, w_m)) \equiv \nabla\!\!\!\!\nabla^{-t}((C_1, -w_1), \ldots, (C_m, -w_m)) \tag{8}$$

$$\nabla\!\!\!\!\nabla^{=t}((C_1, w_1), \ldots (C_m, w_m)) \equiv \nabla\!\!\!\!\nabla^t((C_1, w_1), \ldots, (C_m, w_m) \sqcap \nabla\!\!\!\!\nabla^{\leq t}((C_1, w_1), \ldots (C_m, w_m) \tag{9}$$

Weighted concepts and preferences

Given a concept combination $\mathtt{C}$ and a model $I = (\Delta^I, \cdot^I)$, we can define an ordering over the instances as follows: for every d, d',

$$d \succeq_{\mathtt{C}} d' \Leftrightarrow v^I_{\mathtt{C}}(d) \geq v^I_{\mathtt{C}}(d') \tag{10}$$

Moreover, an ordering on the object names that depends on the context $\mathcal{K}$ is introduced by:

$$a \succeq_{\mathtt{C},\mathcal{K}} b \Leftrightarrow v^{\mathcal{K}}_{\mathtt{C}}(a) \geq v^{\mathcal{K}}_{\mathtt{C}}(b) \tag{11}$$

Weighted concepts and preferences

For any $\mathtt{C}$ and $\mathcal{K}$, the ordering $\succeq_{\mathtt{C},\mathcal{K}}$ satisfies (rephrasing of) axiom 1 and 2:

- Axiom 1: if $\{C \textit{ in } \mathtt{C} \textit{ s.t.} \mathcal{K} \models C(x)\} = \{C \textit{ in } \mathtt{C} \textit{ s.t. } \mathcal{K} \models \mathtt{C} \textit{ s.t.} C(y)\}$ then $x \sim_{\mathtt{C},\mathcal{K}} y$.
- Axiom 2: for any x, y in X and any $\mathtt{C}$, $\mathtt{C}'$, s.t. $\mathtt{C} \subseteq \mathtt{C}'$, if no C in $\mathtt{C}'$ is true of x and y, then $x \succeq_{\mathtt{C},\mathcal{K}} y$ iff $x \succeq_{\mathtt{C}',\mathcal{K}} y$.

So we can rephrase in this context Theorem 1 of (Dietrich and List, 2013).

Ranking sets of reasons

A weighted concept combination $\mathtt{C}$ also induces an ordering on the (consistent) sets of reasons.

Given $C_i \subseteq X$, $S = \{C_1, \ldots, C_m\}$, $\succeq'_{\mathtt{C}} \subseteq \mathcal{P}(S) \times \mathcal{P}(S)$: for A, B in $\mathcal{P}(S)$:

$$A \succ'_{\mathtt{C},\mathcal{K}} B \Leftrightarrow v_{\mathtt{C}}^{A(d)}(d) \geq v_{\mathtt{C}}^{B(d)}(d) \tag{12}$$

Where, $d \in X$,
$A(d) = \{A_1(d), \ldots A_l(d)\}$, for $A_i \in A$,
$B(d) = \{B_1(d), \ldots B_h(d)\}$, for $B_i \in B$.

Ranking sets of reasons. Expressivity

Given a set of reasons $S = \{C_1, \ldots, C_m\}$, if the C_i are atomic concept names and the Tbox of $\mathcal{K}$ is empty, then every function $\mathcal{P}(S) \to \mathbb{R}$ can be represented by means of a concept combination $\mathtt{C}$.

Theorem

For every $f : \mathcal{P}(S) \to \mathbb{R}$, there exists a concept combination $\mathtt{C}$ such that for every $A \in \mathcal{P}(A)$,

$$f(A) = v_{\mathtt{C}}^{A(d)}(A)$$

The argument adapts the representation of utility function by means of goal bases in (Uckelman et al, 2009).

E.g. the additive weighing of reasons in (Dietrich and List, 2013).

Note that, in the case of a non-trivial Tbox or complex concepts in S, not every utility function $f : \mathcal{P}(S) \to \mathbb{R}$ is legitimate: e.g. if $\mathcal{K} \models C \equiv D$, then $v_{\mathtt{C}}^{\mathcal{K}}(\{C\}) = v_{\mathtt{C}}^{\mathcal{K}}(\{D\})$.

Examples

Given a set of alternatives X, and preferences based on $\mathtt{C}$, the $\mathbb{W}$ operators allow for expressing a number of facts about the preference ordering, e.g.:

- $\mathbb{W}^{max}((C_1, w_1), \ldots (C_m, w_m))$: the best alternatives of the preference ordering.
- $\mathbb{W}^{min}((C_1, w_1), \ldots (C_m, w_m))$: the worst alternatives of the preference ordering
- $\mathbb{W}^{t}((C_1, w_1), \ldots (C_m, w_m))$: the alternatives that score exactly t.

Aggregating reason-based preferences. Setting the problem.

- Given a set of N agents, a set of alternatives X, an agenda of reasons S, each agent has preferences on $\mathcal{P}(S)$ represented by $\mathtt{C}_i$ and information represented by $\mathcal{K}_i$.
- Each $\mathtt{C}_i$ induces an ordering $\succ'_i$ (or a utility function u_i) on an agenda of reasons as well as an ordering $\prec_i$ on alternatives.

Aggregation problem:

$$\begin{array}{cccc} \mathcal{K}_1 & \mathtt{C}_1 & \succ'_1 / u_1 & \succ_1 \\ \mathcal{K}_2 & \mathtt{C}_2 & \succ'_2 / u_2 & \succ_2 \\ & & \ldots & \\ \mathcal{K}_n & \mathtt{C}_n & \succ'_n / u_n & \succ_n \end{array}$$

Aggregating reason-based preferences. Open Problems:

$$
\begin{array}{cccc}
\mathcal{K}_1 & \mathtt{C}_1 & \succ_1' / u_1 & \succ_1 \\
\mathcal{K}_2 & \mathtt{C}_2 & \succ_2' / u_2 & \succ_2 \\
 & & \cdots & \\
\mathcal{K}_n & \mathtt{C}_n & \succ_n' / u_n & \succ_n
\end{array}
$$

- Aggregate the $\mathcal{K}_i$ (Judgment Aggregation) first, then agree on some $\mathtt{C}$?
- Agree on some $\mathtt{C}$, then agents evaluate $\mathtt{C}$ from their point of view $\mathcal{K}_i$?

Examples:

- *Utilitarian, cardinalist*: Aggregate the u_i, then associate a $\mathtt{C}$ and the preferences over alternatives.
- *Utilitarian, ordinalist*: Aggregate the $\succ_i'$, then associate a $\mathtt{C}$ and preferences over alternatives.
- *Deliberativist* (?):
- *Explaining the collective choices*: Aggregate the preferences $\succ_i$, find a $\mathtt{C}$ that "rationalises" $\succ$.

Conclusion and future work

- We used a well-established setting in Knowledge Representation to discuss weighted combinations of reasons.
- We associated preferences with the (weighted) satisfaction of a number of reasons.
- We proposed how to rank sets of reasons.
- We proposed a setting to phrase reason-based preference aggregation.

Future work

- Besides the many needed fixings, make sense of reason-based preference aggregation.

References

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