Level statistics of the Gaussian $\beta$ ensemble

Wouter Buijsman (UvA)
Vladimir Gritsev (UvA), Vadim Cheianov (UL)

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Setting the stage

- Integrable systems
- Thermal systems
- Non-thermal dynamics
- Poissonian level statistics
- Conserved quantities
- Wigner-Dyson level statistics
- No conserved quantities
- Thermal dynamics

Wouter Buijsman

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Thermal system spectrum (no time-reversal symmetry)

$P(s)$

Level spacing $s_i$
Dyson’s threefold way

Thermal Hamiltonians come in 3 symmetry classes:
(Dyson, J. Math. Phys. 1962)
- Diagonalizable by orthogonal transformations ($\beta = 1$)
- Diagonalizable by unitary transformations ($\beta = 2$)
- Diagonalizable by symplectic transformations ($\beta = 4$)

Locally, level statistics are described by the Gaussian $\beta$ ensemble

Gaussian $\beta$ ensemble

Levels $\{x_1, x_2, \ldots, x_N\}$ are distributed as

$$P(x_1, x_2, \ldots, x_N) \sim \prod_{i=1}^{N} e^{-\beta \frac{x_i^2}{2}} \prod_{1 \leq i < j \leq N} |x_i - x_j|^{\beta}$$
integrable systems

Poissonian

\[ P(s) = \exp(-s) \]

thermal systems

Wigner-Dyson

\[ P(s) \approx \frac{\pi}{4} s^2 \exp\left(-\frac{\pi}{4} s^2\right) \quad [\beta = 1] \]

\[ P(s) \approx \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi} s^2\right) \quad [\beta = 2] \]

\[ P(s) \approx \frac{26144}{729\pi^3} s^4 \exp\left(-\frac{64}{9\pi} s^2\right) \quad [\beta = 4] \]
Motivations to study the Gaussian $\beta$ ensemble for generic $\beta$:
- Poissonian level statistics are obtained for $\beta \to 0$

Central questions:
- How do the level statistics for $0 \leq \beta \leq 1$ look like?
- What is the physical meaning of this interpolation?

Relevance:
- Statistics apply very well to many-body localization transition (Luitz, Laforencie, Alet, PRB 2015)
- Interpolation has a thermodynamic interpretation (P.J. Forrester, Log-Gasses and Random Matrices, 2010)
Level spacing distribution for generic $\beta$
Summary, conclusions and outlook

Summary:
- We study statistics of the Gaussian $\beta$ ensemble for generic $\beta$
- Aiming to interpolate between ‘P’ and ‘WD’ level statistics

Conclusions:
- We have numerically obtained the interpolating statistics
- Results fit to the many-body localization transition (not shown)

Outlook:
- What is the interpretation of $\beta \notin \{1, 2, 4\}$?
- How does the interpolation relate to other interpolations?
  (Shklovskii, Shapiro, Sears, Lambrianides, Shore, PRB 1993)
- Does the interpolation fit to other models? (arXiv:1802.08827)