What Gaia can reveal about the matter distribution in the Milky Way

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Abstract

**Context.** With the goal to accurately map about a billion of the Milky Way stars, the astrometric satellite Gaia was launched in December 2013. Its high precision and sensitivity will lead to better understanding of the Galactic structure and evolution. Also, it will be possible to probe the matter distribution in the Galaxy.

**Aims.** To study how well the Galactic matter distribution can be determined from Gaia data, using a direct application of the Jeans equations.

**Methods.** An expression for mass density applicable to collisionless gravitational systems can be found by combining the second Jeans equations with Poisson’s equation. Through orbit integration of a few million stars in a potential model of the Milky Way, astrometric data were simulated. Taking into account extinction using a smooth model based on hydrogen observations, measurement errors expected from Gaia were generated for the chosen tracer stars (M-giants). Applying a grid covering the region of the model set by the magnitude limits of Gaia, the mass density was estimated in every grid-bin and thus resolution was achieved.

**Results.** I find that within the region limited by $5.5 \lesssim R \lesssim 12$ kpc and $|z| \lesssim 200$ pc, the mass density can be estimated with better than 35 % accuracy. Within the smaller region limited by $6 \lesssim R \lesssim 9$ kpc and $|z| \lesssim 50$ pc, the estimation error has decreased to a few percent and in the Solar neighbourhood it will be possible to probe the matter distribution with about one percent precision. Both the regions mentioned cover a large part of the Perseus spiral arm and spans over different locations where the individual components of the Galaxy are prominent.

**Conclusions.** The extensive regions probed with this method make it possible to determine the large-scale structure of the Milky Way, including e.g., more accurately determined radial and vertical scale lengths. It will be possible to improve the estimation of the local mass density by about a factor ten compared to the current values. The precision of the method allows to test different scenarios for the distribution of dark matter in the disk and elsewhere in the Galaxy.
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**Populärvetenskaplig sammanfattning**

Att galaxer är uppfyggda av stjärnor, gas, stoft och mörk materia är numera ett välkänt faktum. Hur de bildades och utvecklades till så otroligt olika objekt som idag syns på himlavelvet är däremot fortfarande ganska okänt. Genom att undersöka Vintergatans stjärnor har man länge försökt ta reda på Galaxens usprung och historia med förhoppningen att då också kunna förklara hur andra galaxer kommit till.


I mitt examensarbete har jag undersökt om det med Gaias kommande stjärnkatalog kommer att vara möjligt att kartlägga Vintergatans massfördelning. Jag har använt mig av en enkel metod som är baserad på i vilka banor testpartiklar påverkade av gravitationen skulle röra sig. I mitt fall motsvarar testpartiklarna stjärnor som rör sig i Vintergatans gravitationsfält. Genom att simulera stjärnornas banor i en enkel modell av Vintergatans kraftfält har jag skapat data i form av stjärnors positioner och hastigheter. Dessa data har jag sedan modifierat för att de ska likna de observationer Gaia kommer att genomföra. En stor fördel med att använda sig av simulera data är att det går att jämföra direkt med den antagna modellen hur bra den uppskattade massdensiteten är och på så sätt är det lättare att utveckla metoden.


Denna möjlighet att mäta massdensiteten i stora områden i framförallt Vintergatans skiva kommer leda till att geometriska parametrar såsom skivans skalhöjd och skallängd kommer kunna bestämmas.

## Contents

A Full derivation of expression for mass density 48
   A.1 Derivation of the continuity equation 48
   A.2 Acceleration in cylindrical coordinates 48
   A.3 From collisionless Boltzmann equation to the second Jeans equations 49
   A.4 Deriving the mass density expression 50

B Finding values for $\sigma_0$ 52

C Star investigation with the Besançon model 54

D Interpretation of the extinction model of Amôres & Lépine (2005) 56

E Including spiral arms 58
   E.1 Potential model with spiral arms 58
   E.2 Simulation results for spiral arms 61
      E.2.1 perfect_FBJ13 61
      E.2.2 xcut5phom_FBJ13 61
   E.3 Discussion and conclusion 65

F Wordlist 66
Chapter 1

Introduction

In this master’s thesis, it is investigated to what extent the Galactic matter distribution can be determined using data coming from the astrometric satellite Gaia. This is done through a direct application of the Jeans equations on simulated data.

**The Galactic matter distribution**  Formation and evolution of galaxies are areas currently not very well understood. A clue towards better understanding in these fields could be found by more accurately probing the current Galactic structure and matter distribution. This because knowledge of the matter distribution is a prerequisite for analysing all sorts of dynamical phenomena (that depend on the Galactic potential) which thus shape the evolution of the Galaxy. Unknown dynamical mechanisms within galactic structure are for example spiral arms and bars [Binney & Tremaine, 2008].

The Galactic matter distribution is connected to the Galactic potential through Poisson’s equation, Eq. (3.9). Knowing the potential would be useful to for example back-trace trajectories of stars and find common birth-places of stars or origin of runaway and hypervelocity stars [Perryman, 2009].

Additionally, a map of the Galactic mass density would, together with a comparison to the visible matter distribution, enable putting limits on or even probing the dark matter content and distribution in the Milky Way. This, in turn, would provide information about how dark matter interacts and forms clumpy structures of varying size. It could prove or disprove the existence of suggested dark matter structures such as a dark disk in which the Galactic disk would be embedded and establish or put an upper limit on the dark matter concentration at different Galactic regions [Bruch et al., 2009].

**The satellite Gaia**  The 19th of December 2013, the astrometric satellite Gaia was launched from ESA’s launch site in Kourou, French Guyana. The goal of Gaia is to create a three-dimensional map of our home-galaxy, the Milky Way, by observing about a billion stars all over the Galaxy. That Gaia is an astrometric satellite means that the observations of the stars will contain measurements of positions and velocities of stars.

Gaia was constructed by ESA after the great success of its predecessor Hipparcos, and this with the purpose to with higher precision observe more stars and out to larger distances. The astrometric data in the Hipparcos catalogue include 118 218 stars measured with a precision of $1 \pm 2$ mas. The stellar map of the Solar neighbourhood created by Hipparcos helped to, amongst other things, provide the most accurate confirmation at the time of Einstein’s prediction of the effect of gravity on starlight and to improve the knowledge about the distributions, motions and ages of stars. In particular, it gave the first firm estimate of the average mass density in the Solar neighbourhood [Perryman et al., 1997a; Crézé et al., 1998; Holmberg & Flynn, 2000]. Hipparcos also defined the most accurate optical realisation to date of the International Celestial Reference System (ICRS), which is the system of coordinates (right ascension, declination) used by astronomers for identifying and tracking astronomical objects.

The Gaia satellite will measure with micro arcsecond precision [Lindegren, 2005] about a billion stars in the Milky Way [Lindegren, 2005; Robin et al., 2012]. These data is anticipated to help understanding the Galactic structure and through this Galactic formation and evolution. Also, the set of data is thought to contain information about, amongst other things, exoplanets, present and past star formation rate (SFR) and the role of binary stars in stellar populations such as clusters [Perryman et al., 1997a].

The improvements in Gaia compared to its predecessor Hipparcos lies mainly in the higher accuracy and the possibility to detect weaker light sources. The higher accuracy comes as an effect of several things. Firstly,
Gaia’s mirrors are bigger (about $1.46 \times 0.51$ m compared to Hipparcos circular mirror with diameter of 30 cm), which allow more light to enter the telescope. Secondly, the detectors of Gaia are more sensitive as the used technique is CCDs and not photomultiplier tubes as on Hipparcos. Apart from the photomultiplier tubes being less efficient than CCDs are, they also had to be focussed on individual stars, while CCDs can observe the whole entering field of view. This means that Gaia can scan the whole sky, which in turn enhances the amount of objects observed per unit time and the amount of times an object can be observed. In this way the accuracy of the final results are enhanced. Thirdly, the modulating grid in the detection system of Hipparcos prevents some of the incoming light to enter the detector. This will not happen with the CCDs of Gaia (Lindegren 2005; Perryman 2009). The mission time of Gaia is also longer than for Hipparcos with five years compared to three.

Summary of work performed The datasets used in this work are simulated as Gaia is in the process of gathering data. Simulating data is actually suitable as the properties of the data are already known and can be used to verify the accuracy of the developed method. The data consist of positions and velocities of stars modified to seem more realistic by adding measurement errors from the satellite after having estimated the extinction.

The stellar coordinates are found from a model of the Galactic potential, which enables actually knowing the matter distribution in the model through Poisson’s equation, Eq. (3.9). Using an expression for the mass density derived from the Jeans equations, thus containing terms with positions, velocities and number densities, the mass density in the potential model is derived using the stellar data.

The derived method is tested first on a set of very accurate data to check that it is actually working the expected way. After the verification, the dataset is step by step modified to seem more like data which would actually come from Gaia, i.e., including measurement errors and extinction.

Figure 1.1 illustrates the structure of the work. The blue path signifies how the model is built, the red how the functionality of it can be verified and the gray part shows the future of incorporating real data in the process.

Similar types of work The method outlined above has been used in smaller scale to estimate the local matter and dark matter densities (Holmberg & Flynn 2000; Garbari et al. 2011). The difference this work has compared to these estimations is that in this case it will be possible to create a map with resolution and not just a single number as the region Gaia will probe extends over a large part of the Galaxy.

Alternative approaches to estimate the Galactic matter distribution are described by Binney (2005), who include then Schwarzschild modelling, torus modelling and Syer-Tremaine modelling.

Outline of work performed This report is structured in the following way. In Chapter 2 the background necessary for carrying out the investigation is given. This includes a closer look at how the satellite Gaia works and an overview of what is known about the matter distribution in the Galaxy.

In Chapter 3 an expression for the mass density is derived and it includes only terms dependent on the observable parameters position, velocity and number density of stars. The expression is derived from the Jeans equations and has not been encountered in the literature by the author.

Chapter 4 explains how the simulated Gaia dataset is created. This includes the development from perfect accuracy to modified data with measurement errors and extinction. The chapter is finished by explaining how the expression derived in Chapter 3 is used with the simulated data.

The results of the estimated mass density in the potential model of the Galaxy are given in Chapter 5. This is done by showing density plots of the estimated mass density and the difference from the true mass density. The results are given for each step of the evolution of the data, from perfect precision to data with measurement errors and extinction added.

Finally, in Chapter 6 the conclusions are drawn and discussed.
Simulated data \((R, \theta, z, u, v, w, n)\)

Stellar path integration

Choice of potential, \(\Phi\)

Find \(\rho\) through Poisson’s equation

Simulated data \((R, \theta, z, u, v, w, n)\)

Add measurement errors

Estimate \(\rho\)

Comparison of \(\rho\)

Add extinction and regenerate measurement errors

Data from Gaia

Figure 1.1: A flow chart of the methodology used in this thesis for testing the mass density estimation. It begins with a potential model of the Galaxy, \(\Phi\). The mass density, \(\rho\), can then either be calculated directly from the Poisson’s equation, Eq. (3.9) (red path), or estimated by letting stars orbit in the potential and apply a mass density estimation method on the generated data (blue path). Here, the dataset is made more realistic by adding measurement errors and extinction. In the future, it might be possible to apply this technique to real Gaia data (gray box), but then, as the potential is not known, but rather desired to be found, the comparison cannot be performed.
Chapter 2

Background

This chapter handles necessary background material needed for Chapter 4 where the method used in this work is described. Section 2.1 describes the satellite Gaia and Sect. 2.2 gives an overview of what is known about the matter distribution in the Galaxy.

2.1 Gaia

The recently launched satellite Gaia is very important for this work. Therefore, the following section describes what the main goals are, why it will be of importance to science and how it observes.

2.1.1 Scientific goals

The goal of Gaia is to create a three-dimensional map of the Milky Way galaxy by observing stars out to very large distances. In total, Gaia is estimated to observe roughly a percent of the stars in the Galaxy, which corresponds to about a billion stars (Robin et al., 2012).

With this three-dimensional star map, the stellar populations in the different Galactic components can be examined as the dynamics and physical properties of individual stars will be included in the survey. Additionally, tests of general relativity will be possible to perform as Gaia also will observe a large quantity of objects \(10^5 - 10^6\) in the Solar System (Robin et al., 2012; Perryman, 2012; Perryman et al., 2001). With the astrometric measurements of the stars, exoplanets will be observed in the Gaia data from wobbly stellar motion caused by the gravitational pull of planet(s) (Perryman, 2012). Another goal which is of relevance for this project is to probe the Galactic matter distribution and investigate the dark matter content and distribution (Perryman et al., 1997a).

2.1.2 Motivation for the mission

In 1989 the predecessor of Gaia, the ESA mission Hipparcos, was launched. The Hipparcos catalogue includes 118 218 stars (roughly a hundredth of a percent of Gaia’s capability), mostly located in the Solar neighbourhood. They were measured with a precision of 1 – 2 mas, which means the precision of the measurement of the angular position on the sky (Perryman et al., 1997b; Perryman & ESA, 1997). With this catalogue came great achievements such as improved ICRS, a more precise confirmation of Einstein’s prediction of the effect of gravity on light and an estimation of the mass density in the Solar neighbourhood. This estimation was found to be \(\rho_0 = 0.102 \pm 0.010\, M_\odot\, pc^{-3}\) (Holmberg & Flynn, 2000), which is lower than the former estimations of \(\rho_0 = 0.18\, M_\odot\, pc^{-3}\) (Binney & Tremaine, 1987; Burton & Gordon, 1978).

From the great success of Hipparcos, already in the beginning of the 1990’s, the idea of Gaia started to form (Lindgren & Perryman, 1995). With improved detector sensitivity and increased satellite size to enable more precise measurements, Gaia would be able to reach regions in the Milky Way that ranged to regions 100 times more distant than Hipparcos capabilities. Instead of covering only the Solar neighbourhood as Hipparcos did, Gaia, with its developed precision and sensitivity, would be able to reach a significant fraction of the entire Galaxy. This would mean that dynamical mechanisms, like the spiral arms, could be examined along with different stellar populations. These, together, might unveil the history and formation of the Milky Way galaxy.
2.1 Gaia

As our galaxy is not unique, such knowledge would be revolutionising for the field of galaxy formation and evolution in general, which is the main scientific purpose of the satellite \cite{Perryman1997a, Perryman2001}.

2.1.3 Astrophysical instruments

Since about a month after the launch in December 2013, Gaia is orbiting the Sun together with the Earth, but from the second Lagrange point (L2) located roughly a percent of an AU behind the Earth on the Earth-Sun line. As the second Lagrange point is a metastable gravitational point\(^1\), minimal energy has to be added to keep the satellite on the simultaneous orbit around the Sun. This fact, together with the advantageous situation that the light of the Sun and the Earth (the two main light sources in the vicinity) will always be in the same region of the sky and thus easier to obscure, L2 was chosen to be the ultimate location to put Gaia.

At its observing point, Gaia spins around its own axis, rotates around the Sun-Earth axis and orbits the Sun. These motions together let Gaia scan the whole sky and the plan is to do so during the whole mission time of five years. Being equipped with two telescopes, Gaia can observe two fields of view simultaneously. This is the way Gaia achieves its high precision as is explained further and more technical in Sect. 2.1.4.

Where the starlight hits the detector, Gaia has three scientific instruments and their set up is seen in Fig. 2.1. Their main task is to together accurately find the positions, velocities and spectral types of the observed stars. More precisely how this works is briefly described below\(^2\).

2.1.3.1 Astrometric Field (AF)

The main instrument of Gaia is the Astrometric Field (AF). It consists of 62 CCDs oriented so that each star passes nine (or eight if it happens to be in the middle regions) CCDs every time it is observed as is seen in Fig. 2.1. The Astrometric Field observes in the Gaia \(G\) band, which is close to the Johnson-Cousins visible \(V\) band, but broader over the red wavelengths. This allows Gaia to observe stars which are bright in red and even infrared. By polynomial fitting \cite{Jordi2010} found a relation between the band determining the Gaia \(G\) magnitude and the Johnson-Cousins \(V\) magnitude. It is written as

\[
G = V - 0.0257 - 0.0924 \cdot (V - I_C) - 0.1623 \cdot (V - I_C)^2 + 0.0090 \cdot (V - I_C)^3, 
\]  

(2.1)

where the colour index \(V - I_C\) is incorporated because of the earlier mentioned wider shape of the Gaia \(G\) band.

The CCDs of the Astrometric Field are sensitive enough to detect stars with \(\mu\)as precision if they have a lower magnitude than \(G = 20\) mag \cite{Lindgren2005}. If \(G < 6\) mag another limit is hit as then the CCDs

---

\(^1\)A point where the gravitational force from the Sun and Earth equals the centrifugal force of a body moving with the point.

\(^2\)The information in this section is generally found from ESA at the Gaia webpages \url{http://gaia.esa.int/} and \url{http://www.cosmos.esa.int/web/gaia}.
in the Sky Mapper are saturated and the star is not observed by either of the instruments. (The Sky Mapper is seen in Fig. 2.1 to be the first two columns of CCDs and it has the task to register which stars should be observed.)

Using the Besançon model Robin et al. (2012) estimate the amount of observable stars within these magnitude limits to be about 1.1 billion, distributed all over the Milky Way. Taking into account the way Gaia scans the sky, each of these stars will be observed roughly 70 times after the whole mission time.

2.1.3.2 Radial Velocity Spectrometer (RVS)

The Radial Velocity Spectrometer (RVS) on-board Gaia splits the starlight into spectra in the narrow near infrared wavelength region of 847 – 874 nm. The light splitting happens when the incoming light passes through an optical module containing a grating plate and lenses. After the passage, the light is projected over the 12 CCDs dedicated to the RVS (see Fig. 2.1). In the wavelength region of the Gaia $G_{RVS}$ band resides the calcium triplet, which is three transition lines of ionised calcium very common to find in stars. Measuring the Doppler shift of the calcium triplet, the velocity of the star in the line of sight (also called radial velocity) can be found. Because of the location of the calcium triplet, the band in which the RVS observes does not need to be very broad. A relation to the Johnson-Cousins system and the Gaia $G_{RVS}$ magnitude is found from

$$G_{RVS} = V - 0.0119 - 1.2092 \cdot (V - I) + 0.0188 \cdot (V - I)^2 + 0.0005 \cdot (V - I)^3,$$

also via polynomial fitting by Jordi et al. (2010).

For the split starlight to constitute a reasonable spectrum, a minimum amount of photons are needed. As a full spectrum is required to make the radial velocity measurement, the limiting magnitude for the RVS is lower than for the AF. Radial velocities can be found in the RVS for stars with a magnitude lower than $G_{RVS} = 16$ mag and higher than $G = 6$ mag. This limit Robin et al. (2012) find allows for roughly 150 million radial velocity estimates to be made by Gaia. From the size of the RVS and scanning law of Gaia, it is estimated that each of these stars can be observed about 40 times by the RVS.

2.1.3.3 Blue and Red Photometers (BP/RP)

The photometers on Gaia are used to classify the observed stars and finding their properties such as effective temperature, colour, surface gravity etc. This is done by dividing the starlight into a red and a blue part using filters and combine with the astrometric measurements. In the focal plane (as seen in Fig. 2.1) the blue and the red photometers have each a column of nine CCDs over which each of the stars observed by the Astrometric Field passes as well.

In this work, the photometers are not directly considered as the astrometric measurements are the relevant data to the investigation.

2.1.4 Astrometric observations

The many observations the Astrometric Field makes of every star throughout the mission time can be used to find the path the star travels on the sky by fitting a trajectory to the observations. As an example, Fig. 2.2 shows the procedure done for a star observed by Hipparcos. By fitting a trajectory to the observed positions, the uncertainty in position measurements decreases with a factor $\sqrt{n}$ where $n$ is the number of observations included in the fit.

As is seen in Fig. 2.2 the star moves in a spiral pattern on the sky. This phenomenon comes from the satellite moving together with the Earth around the Sun and the star seems to move compared to the background. The angular size of the loops in the spiral pattern is called the star’s parallax and is connected to the distance to the star through trigonometry in the following way

$$d \ (\text{pc}) = \frac{1 \ (\text{AU})}{\tan \varpi \ (\text{as})} \approx \frac{1 \ (\text{AU})}{\varpi \ (\text{as})},$$

(2.3)

The purpose of double telescope in Gaia is to make the measurements of the parallax more robust as the position of the star not only is compared to stars in its close surroundings as seen by one telescope, but also compared to stars in a complete different direction. This verification enables to determine the parallax so-called absolutely (for more technical description of this procedure, see Lindegren, 2005). The parallax is then estimated
with an uncertainty limited by instrumental properties and the size of the point spread function. From Gaia Science Performance webpage at ESA\footnote{http://www.cosmos.esa.int/web/gaia}, the parallax uncertainty (standard deviation) is written (in $\mu$as) as

$$
\sigma_\varpi = (9.3 + 658.1 \cdot z + 4.568 \cdot z^2)^{1/2} \cdot (0.986 + (1 - 0.986) \cdot (V - Ic)) \cdot (0.986 + (1 - 0.986) \cdot (G - 15)),
$$
\label{eq:parallax_uncertainty}

where the colour index is involved as a redder star gives rise to a larger point spread function and thus a more imprecise measurement. The uncertainty in the position measurement is correlated to the trajectory as the fit makes this measurement much better. Therefore it can be found from the parallax uncertainty, but it will not be used in this work as it is found to be negligible (see Sect. \ref{sec:Gaia_performance}) and therefore the relation is not given here.

The trajectory fit is not only used to find the angular position of the stars on the sky and the distance to them, but also to find their motion on the sky. This is done by subtracting the parallactic motion from the fitted trajectory and averaging the distance moved on the sky over the mission time. Because of the correlation between these different measurements through the fitted trajectory, also the uncertainty in the measurement of the proper motion (denoted $\mu$ and signifying the motion on the sky) is connected to the uncertainty for the parallax measurement. This in the following way

$$
\sigma_\mu = 0.526 \cdot \sigma_\varpi, 
$$
\label{eq:proper_motion_uncertainty}

where $\sigma_\mu$ then is given in $\mu$as/yr$^3$. From Eqs. \eqref{eq:parallax_uncertainty} and \eqref{eq:proper_motion_uncertainty} it can be found that properties inducing larger astrometric measurement errors belong to weakly shining and redder stars.

In the coming Gaia catalogue, the data will consist of the average proper motion and parallax together with the noted position of each star in roughly the mid-mission time. The mid-mission time is chosen as the location of the other observations then make it the most accurate position estimate.

The radial velocity measurement is (as mentioned in Sect. \ref{sec:Gaia_radial_velocity}) not found by the same instrument, but through Doppler shift of transition lines in the Radial Velocity Spectrometer (RVS). The measurement uncertainty for the RVS is therefore not correlated to the parallax uncertainty, but still depends on the size of the point spread function, that is colour, and instrumental limitations such as magnitude limits of CCDs etc. What
Figure 2.3: Dispersions to expect in the measured parallax ($\varpi$), proper motion ($\mu$) and radial velocity ($v_r$) for different spectral types, first main sequence stars (V), then giant stars (III). Dots correspond to a distance of 1 kpc, squares to 3 kpc, rings to 10 kpc and triangles to 30 kpc. Relative parallax uncertainty (second panel) could be found as the parallax is the same for all stars at the same distance. This is not the case for proper motion or radial velocity, which are both individual to each stars.
also affects the radial velocity measurement is the shape of the transition lines. With a thin shape, it is easier to pinpoint the Doppler shift and thus the uncertainty decreases. Particular spectral types with this feature are giant stars. Their spectral lines are thin primarily because of the surface gravity of the star is low, which implies lower pressure and less pressure broadening of the lines. Combining these effects, the uncertainty in the measurement of radial velocity is given as

\[ \sigma_{v_r} = 1 + b \cdot e^{a(V - 14)}, \]  

(2.6)

where \( \sigma_{v_r} \) is given in km/s and \( a \) and \( b \) are constants which contain information about magnitude, colour and linewidth.\(^4\) For more information about how the constants are found see Appendix B.

The varying luminosity and colour of different spectral types give them different uncertainties in the observations. In Fig. 2.3, the uncertainty in parallax, proper motion and radial velocity of different spectral types are given for distances of 1, 3, 10 and 30 kpc.

### 2.1.5 Distances from photometry

What primarily is used to find distances to stars in this work is the astrometric way via parallaxes. An alternative way to estimate the distance to a star is via photometry. For Gaia, in most cases, the photometrically estimated distance is not as accurate as the astrometrically estimated distance, but for very distant stars or stars gravely perturbed by extinction, the parallax might be so badly estimated that photometry becomes useful.

When observing a star in a specific band \( X \), the apparent magnitude, \( m_X \), is correlated to the absolute magnitude \( M_X \) and the extinction \( A_X \) together with the distance \( d \) in the following way

\[ m_X = M_X + 5(\log_{10} d - 1) + A_X, \]  

(2.7)

where the distance is given in pc and the rest of the terms in magnitudes. The distance modulus \( \mu \) (this is not proper motion, the notation is unfortunate) can then be derived from Eq. (2.7) to be

\[ \mu = m_X - M_X - A_X = 5(\log_{10} d - 1), \]  

(2.8)

which means that if it is found, also the distance can be found. As is seen in Eq. (2.8), to find the distance modulus, both the apparent magnitude, the absolute magnitude and the extinction along the line of sight needs to be found. The apparent magnitude is correlated to the incoming flux of photons, thus it is directly measurable. From the shape of the observed spectral energy distribution (SED), that is the spectrum of the star, the effective temperature, \( T_{\text{eff}} \), and the extinction, \( A_X \), can be inferred. The absolute magnitude is found by placing the star on the Hertzsprung-Russell diagram using the found parameters and probability of location on the diagram. This is simpler for bright, main sequence stars as for their temperature it is by far most common to find stars in the region of bright, main sequence stars. With lower temperatures the probability that the star is going through its main sequence or whether it is located on the giant branch becomes more similar and to determine the type, even a badly estimated parallax can come to use, see Eq. (2.7). After estimating the apparent magnitude, the absolute magnitude and the extinction along the line of sight, Eq. (2.8) is used to find the distance modulus and from that the distance to the star. This method is used in e.g., [Hanson & Bailer-Jones (2014)](http://www.cosmos.esa.int/web/gaia).

The distance modulus is used here as an experiment to investigate the improvement in the results if the distance to very distant stars could be better estimated. Because it is an experiment, the relative measurement uncertainty for distance found from photometry is optimistically estimated to be 20 %,

\[ \epsilon_{\text{ph}} = 0.2, \]  

(2.9)

where the subscript ph refers to photometry. In the case where this is similar to the relative uncertainty from parallax, it would be optimal to calibrate the estimation by using both the photometric and the astrometric estimate. This is done by combining the two normally distributed uncertainties through multiplication with the following result

\[ \epsilon_{\text{tot}} = \left[ \epsilon_{\varpi}^2 + \epsilon_{\text{ph}}^2 \right]^{-1/2}, \]  

(2.10)

where the astrometrically found relative uncertainty is \( \epsilon_{\varpi} = \sigma_{\varpi} / \varpi \) and thus the measurement uncertainty for parallax can be found by multiplying \( \epsilon_{\text{tot}} \) with the true parallax.

\(^4\)This information is found at ESA on the Gaia webpage [http://www.cosmos.esa.int/web/gaia](http://www.cosmos.esa.int/web/gaia), date: 10/5 – 2014.
2.2 Matter distribution in the Milky Way

This section gives a review about the formation theory of the Milky Way, which parts of it are verified and what is left to be explained. To a large extent the information comes from Binney & Tremaine (2008) and this reference is therefore omitted from the text.

2.2.1 Theoretical framework

The Universe is believed to be born through the Big Bang. Following the ΛCDM model, along came the dark matter. Dark matter, assuming it to be WIMPs\(^5\), accumulated in some regions to form halos due to the gravitational force affecting the particles and thus also created potential wells in these regions. As the Universe cooled, the baryonic matter searched for the most favourable place to inhabit and fell into the gravitational potential wells of the dark matter. This matter became visible by clumping together into stars and through this process the first galaxies were formed (White & Rees, 1978; Springel et al., 2005).

The spiral arms and bar visible in many disk galaxies are thought to be dynamical phenomena. According to theory, the spiral arms are density waves which increase the matter density in regions and thus induce star formation (Lin & Shu, 1964). The first part of the spiral arms (in the direction of motion) are thus characterised by galactic gas and dust, followed by star formation as an effect of compressed gas and dust. Because the mass density is increased in the spiral arms, many massive stars are formed and as they have high brightness, the spiral arms become visible. Central bars are developed in disk galaxies due to the rotation and they can be differently prominent depending on for example the content of gas and the mass of the halo. It is an evolving object and rotates with a pattern speed decided by for example the loss of angular momentum (Athanassoula, 2003; Athanassoula et al., 2013).

From the fractal structure of dark matter deriving from clumping due to gravitational forces, differently sized potential wells are assumed to be created. When the baryons seek places with lower energy, they can thus also get caught in a smaller dark matter potential well, which are also called subhalos. These smaller assemblies of matter are assumed to form smaller satellite galaxies orbiting larger galaxies (Diemand et al., 2008; Springel et al., 2008). Following the same cosmological model, also globular clusters should contain dark matter (Peebles, 1984). They however do not generally contain any significant amount of dark matter, which may be due to stripping from the tidal field of the Galaxy (Ibata et al., 2013).

If the orbit of a satellite galaxy is such that it is lead to interact or actually come into contact with the large, central galaxy, the result might be tidal disruption of the satellite galaxy or dynamical processes such as appearing density waves from plunge-through or thickening of the disk from a so-called minor merger (Villalobos et al., 2010; Steinmetz, 2012). It has also been speculated whether thicker dark matter disks do form in the larger galaxy after a minor merger (Read et al., 2008; Lake, 1989).

From accretion or tidal disruption of satellite galaxies, stars are left orbiting the centre of the large galaxy in unusual orbits and populate thus the region above and below the galactic disk (Bullock & Johnston, 2005). During the formation of the galaxy, also globular clusters are formed and set orbiting the centre in all sorts of inclinations (Fall & Rees, 1985). The stars with these inclined orbits constitute what is called the stellar halo. The lonely stars in the stellar halo can also have other origins such as tidal ripping of globular clusters or binaries which have been perturbed by the central supermassive black hole (Hills, 1988).

Elliptical galaxies are thought to be results of so-called major mergers, when two large galaxies collide and merge (Toomre, 1977; Hernquist, 1992; Naab et al., 1999). The thin geometry of the disk galaxies disappears after the merge due to different orientations of their angular momenta, but some angular momentum is expected to remain. Therefore, elliptical galaxies are also thought to have rotation.

2.2.2 Verified theory

In the Milky Way, it is well-known that the main components the disk, bulge and halo exist. A thick disk was discovered in the Milky Way by counting stars (Gilmore & Reid, 1983) and it is estimated to have a scale height of 900 pc, while the scale height found for the thin disk component is 300 pc (found by data from SDSS, Jurić et al., 2008). Jurić et al. (2008) also find that the thick disk has a density of 12 % of the thin disk at the location of the Sun.

The rotation curve of a galaxy shows the circular velocity at different radii. In Fig. 2.4 the rotation curve of the Milky Way is shown, based on the Galactic potential model of Paczyński (1990). The model is adapted

\(^5\)WIMP stand for weakly interacting massive particle and is regarded as the most promising candidate for particle dark matter.
Figure 2.4: Rotation curve of the Milky Way model by Paczyński (1990) illustrated by a thick, solid, black line. The rotation curves for the individual components in the model are represented by a thin solid line for the bulge, a dashed line for the disk and a dotted line for the halo.

To fit the observed rotation curves described by Burton & Gordon (1978); Binney & Tremaine (1987).

Spiral arm investigations show that the Milky Way has spiral arms, four detected in optical and two in the K-band (Taylor & Cordes, 1993; Georgelin & Georgelin, 1976; Drimmel, 2000). Two of them seem to be more prominent than the rest (Drimmel, 2000), therefore it is plausible that the Milky Way looks very similar to a grand-design galaxy if seen from the outside.

The central bulge was first suggested to be bar-shaped by de Vaucouleurs (1964). Blitz & Spergel (1991) claim having observed the bar shape through 2.4 μm observations of the Galactic Centre, but the over-all accepted evidence for the existence of a central bar in the Milky Way comes from the COBE satellite with its DIRBE instrument (Dwek et al., 1995). In this survey the observed asymmetry of the bulge is interpreted as one edge of the bar being located closer to the Sun than the other. The bar of the Milky Way is estimated to be located with an angle compared to the Galactic centre-Solar system line of 20° (Wang et al., 2012; Drwe et al., 1995; Binney et al., 1997; Gerhard, 2002; Babusiaux & Gilmore, 2005; Binney & Tremaine, 2008) and has a half length of ∼3 kpc (Binney & Tremaine, 2008; Gerhard, 2002).

Ibata et al. (2001) connect overdensities in the SDSS data to tidal streams being ripped off the Sagittarius dwarf galaxy, a satellite galaxy orbiting the Milky Way, and thus donating stars to the Galactic halo. The total halo of the Milky Way (including dark matter) is thought to initially have been of prolate shape, but has evolved into an oblate shape (Juric et al., 2008; Vera-Ciro et al., 2011).

That satellite galaxies contain a large fraction of dark matter was verified by e.g., Aaronson (1983), Kleyna et al. (2002) and see references therein, while globular clusters have shown no sign of containing dark matter (see Freeman & McNamara, 2006; Moore, 1996) and references therein.

2.2.3 Predicted findings from Gaia

The following expected findings are described in Perryman et al. (1997a).

With the Gaia satellite, the astrometry will be so accurate that the spiral mechanism should be possible to investigate. This, for example in the nearby Perseus arm. Different theories about spiral arm mechanisms are expected to induce different stellar motions, which will be possible to probe with Gaia.

The extremely large stellar catalogue that Gaia will create is thought to hold enough information about the Milky Way bulge stars to track the history of the central bar. Using not only the accurately found positions, but also chemical abundances of a subset of the stars, Gaia hopefully can explain whether the bar was created from disk instability, a merger event etc.

A theory is that large galaxies, such as the Milky Way, get their sizes from merging, either with gas or with smaller satellite galaxies. That minor merging events have taken place in the Milky Way is known, but with which rate and compared to possible gas merging is unknown. By tracking stars, Gaia might be able to determine these numbers.
So far, it has not been possible to determine the dark matter density in the Solar neighbourhood. This would be useful because knowledge about the nature of dark matter itself is predicted to be achieved from such a research. To be able to carry out this research, a precision in the parallax measurements of about 50 µas is predicted to be necessary. This is a number reachable with Gaia.

The dark matter halo of the Milky Way has not been probed. The halo is assumed to be dark matter dominated and therefore by finding properties of the halo, also properties of the dark matter can be found. Establishing the shape of the dark matter halo would be important for galaxy formation and learning about dark matter.

By examining the rotation curve of the Milky Way (for a model, see Fig. 2.4), the dark matter could be probed in the regions beyond the Solar neighbourhood as the stellar motions are sensitive to the amount of surrounding dark matter. No accurate measurement of the Milky Way rotation curve has been made, but with the accuracy of Gaia, this will become easier to do.

Apart from the above mentioned predictions of what Gaia can contribute with, other fields will also benefit from the coming data, such as the physics of stars, finding planets and testing general relativity.
Chapter 3

Theory

This chapter presents the derivation of an expression for mass density containing only terms created from the observable parameters position, velocity and number density of stars. The expression is based on the continuity equation, the collisionless Boltzmann equation and the second Jeans equations. In Sect. 4.2 it is explained how to apply the derived method to simulated data.

In this work, the coordinates most often used are cylindrical, Galactocentric coordinates. Positions are given by radius $R$, azimuth angle $\theta$ and height $z$, while velocities in the corresponding directions are denoted by $u$ in radial direction, $v$ in tangential direction and $w$ in vertical direction. This coordinate system is depicted in Fig. 3.1.

In this chapter, the full derivation will not be displayed. If interest exists, the reader is encouraged to read also Appendix A.

![Figure 3.1: The Galactocentric, cylindrical coordinate system with spatial coordinates $R, \theta, z$ and velocity components $u, v, w$ are used in this work. The Cartesian $(x, y)$-plane is what later will represent the plane of the Galactic disk, where the Sun is situated at $(x, y, z) = (8 \text{ kpc}, 0, 0)$.]

3.1 Finding the second Jeans equations

Considering a system of particles distributed in phase space, their positions and velocities at every specific time $t$ can then be described by the distribution function $f(t, R, \theta, z, u, v, w)$. For a small volume in phase space
at a specific time, the distribution function gives the probable number density of particles in the volume and is thus given in units of $\text{pc}^{-3} \ (\text{km/s})^{-3}$.

The **continuity equation**

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial R} + v \frac{\partial f}{\partial \theta} + w \frac{\partial f}{\partial z} + \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} + \frac{\partial f}{\partial w} = 0 \quad (3.1)$$

describes how the density of particles in a small volume in phase space, co-moving with the particles, remains constant throughout time.

To continue to the collisionless Boltzmann equation from the continuity equation, the case when the acceleration of the particles is caused by a potential is considered. The definition of the potential is then

$$a = -\nabla \Phi, \quad (3.2)$$

where $a$ is the acceleration vector and $\Phi$ the potential. In cylindrical coordinates, this relation leads to the following expressions for the components of acceleration

$$\begin{align*}
\frac{\partial u}{\partial t} &= \frac{v^2}{R} - \frac{\partial \Phi}{\partial R} \\
\frac{\partial v}{\partial t} &= \frac{uv}{R} - \frac{1}{R} \frac{\partial \Phi}{\partial \theta} \\
\frac{\partial w}{\partial t} &= -\frac{\partial \Phi}{\partial z}.
\end{align*} \quad (3.3)$$

Using these expressions for the acceleration in the continuity equation leads to the **collisionless Boltzmann equation**

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial R} + v \frac{\partial f}{\partial \theta} + w \frac{\partial f}{\partial z} + \left( \frac{v^2}{R} - \frac{\partial \Phi}{\partial R} \right) \frac{\partial f}{\partial u} - \left( \frac{uv}{R} - \frac{1}{R} \frac{\partial \Phi}{\partial \theta} \right) \frac{\partial f}{\partial v} - \frac{\partial \Phi}{\partial z} \frac{\partial f}{\partial w} = 0. \quad (3.4)$$

The fact that the acceleration of individual particles can be described by a common potential is what leads to the name collisionless. In a collisionless system the interaction between particles is assumed to be either very weak or very rare so that the potential of individual particles can be neglected. An example of such a system is stars moving in the gravitational force field of a galaxy.

The velocity moments of the collisionless Boltzmann equation are the so called **Jeans equations**. They are achieved by multiplying the equation with $u^i v^j w^k$ and then integrating over $dudvdw$, which will then give a different equation for each combination of $ijk$. The first Jeans equation is the 0th moment which corresponds to $i = j = k = 0$ and gives thus one equation. The second Jeans equations are the first moment equations corresponding to either $i, j$ or $k$ being 1 and the other 0. This gives rise to the following three equations (mean values are denoted with angle brackets $\langle \rangle$)

$$\frac{\partial (n(u))}{\partial t} + \frac{\partial (n(u^2))}{\partial R} + \frac{1}{R} \frac{\partial (n(\langle uv \rangle))}{\partial \theta} + \frac{\partial (n(\langle uw \rangle))}{\partial z} + \frac{n}{R} \left( \langle u^2 \rangle - \langle v^2 \rangle \right) + n \frac{\partial \Phi}{\partial R} = 0 \quad (3.5)$$

$$\frac{\partial (n(v))}{\partial t} + \frac{\partial (n(uv))}{\partial R} + \frac{1}{R} \frac{\partial (n(\langle v^2 \rangle))}{\partial \theta} + \frac{\partial (n(\langle vw \rangle))}{\partial z} + \frac{2n(\langle uv \rangle)}{R} + n \frac{\partial \Phi}{\partial \theta} = 0 \quad (3.6)$$

$$\frac{\partial (n(w))}{\partial t} + \frac{\partial (n(uw))}{\partial R} + \frac{1}{R} \frac{\partial (n(\langle vw \rangle))}{\partial \theta} + \frac{\partial (n(\langle w^2 \rangle))}{\partial z} + \frac{n(\langle uw \rangle)}{R} + n \frac{\partial \Phi}{\partial z} = 0. \quad (3.7)$$

In the above equations, the number density of particles, $n$, is the number of particles per unit volume and thus the distribution function integrated over the velocity components

$$n = \iiint_{-\infty}^{\infty} f \, du \, dv \, dw. \quad (3.8)$$
3.2 Combination with Poisson’s equation

The mass density of the Milky Way galaxy is what creates its gravitational force field, i.e., the potential, and the mass density and potential are therefore connected. The connection is described with **Poisson’s equation**

\[
\nabla^2 \Phi = 4\pi G \rho .
\]

To find the mass density of the Galaxy, the Laplacian of its potential can thus be used. In cylindrical coordinates, the Laplacian of a scalar, which is potential, is written as

\[
\nabla^2 \Phi = \frac{1}{R} \frac{\partial \Phi}{\partial R} + \frac{\partial^2 \Phi}{\partial R^2} + \frac{1}{R^2} \frac{\partial \Phi}{\partial \theta} + \frac{\partial^2 \Phi}{\partial z^2}.
\]

(3.10)

By differentiating the second Jeans equations, Eqs. (3.5), (3.6) and (3.7), they can be used to express the Laplacian given in Eq. (3.10). Inserting this expression in the Poisson’s equation, Eq. (3.9), and solving for the mass density \( \rho \), the following expression for mass density is found

\[
\rho = \frac{1}{4\pi G} \left( \langle u \rangle \frac{\partial^2 \ln n}{\partial R \partial \theta} + \langle v \rangle \frac{\partial^2 \ln n}{\partial R \partial z} + \langle w \rangle \frac{\partial^2 \ln n}{\partial z^2} + \left( \frac{\langle u \rangle}{R} \frac{\partial}{\partial R} + \frac{\partial}{\partial \theta} + \frac{\partial}{\partial z} \right) \partial \ln n + \frac{\partial^2 \langle u \rangle}{\partial R \partial \theta} \right)
\]

\[
\left( 1 + \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} + 2 \frac{\partial^2}{\partial R \partial \theta} + \frac{\partial^2}{\partial R \partial z} + 2 \frac{\partial^2}{\partial R^2} - \frac{1}{R} \frac{\partial}{\partial R} \right) \partial \ln n
\]

(3.11)

The combination of the second Jeans equations to find the expression for mass density in Eq. (3.11) will show itself useful to find the mass density in the Galaxy as the stars can be approximated to particles when considering the vastness of the gravitational potential they are moving in. Therefore, to be able to use the second Jeans equations to probe the matter distribution in the Milky Way, it will be necessary to have information about positions, velocities and number densities of stars in the region desired to be examined.

3.3 Axisymmetry leads to final expression

The expression in Eq. (3.11) is rather long, but through the assumption about axisymmetry and its consequences described below, the final expression will sport a shorter form. Even though the Galactic matter in reality is not axisymmetrically distributed, the assumption of axisymmetry shows itself useful even when considering non-axisymmetric structures (see e.g., Appendix A) and therefore is axisymmetry considered.

**Consequence 1** From axisymmetry follows that the net motion of stars in radial and vertical direction is zero, i.e., \( \langle u \rangle = \langle w \rangle = 0 \). To apply this to the above derived expression for mass density, Eq. (3.11), the elements of the dispersion matrix can be used. The dispersion matrix used in this case is found by the following

\[
D = \begin{pmatrix}
D_{uu} & D_{uv} & D_{uw} \\
D_{vu} & D_{vv} & D_{vw} \\
D_{wu} & D_{wv} & D_{ww}
\end{pmatrix} = \frac{1}{n} \iiint (v - \langle v \rangle)(w - \langle w \rangle)^T f \, d^3v,
\]

(3.12)

which gives that the elements are written the following way when the zero net motions are incorporated

\[
\begin{align*}
D_{uu} &= \sigma_u^2 = \langle u^2 \rangle - \langle u \rangle^2 = \langle u^2 \rangle \\
D_{uv} &= D_{vu} = \langle vw \rangle - \langle v \rangle \langle w \rangle = \langle vw \rangle \\
D_{uw} &= D_{wu} = \langle uw \rangle - \langle u \rangle \langle w \rangle = \langle uw \rangle \\
D_{vv} &= \sigma_v^2 = \langle v^2 \rangle - \langle v \rangle^2 = \langle v^2 \rangle \\
D_{vw} &= D_{wv} = \langle vw \rangle - \langle v \rangle \langle w \rangle = \langle vw \rangle \\
D_{ww} &= \sigma_w^2 = \langle w^2 \rangle - \langle w \rangle^2 = \langle w^2 \rangle.
\end{align*}
\]

(3.13)
Chapter 3: Theory

As is seen above, the squares of the velocity dispersions, \(\sigma^2\), are found on the diagonal of the dispersion matrix.

**Consequence 2** Following from axisymmetry, the Galaxy can be assumed to be static and thus all derivatives with respect to time can be neglected.

**Consequence 3** Again, from axisymmetry follows an independence of the azimuth angle and all derivatives with respect to \(\theta\) can be removed.

Using all three of the mentioned consequences from assuming axisymmetry in Eq. (3.11) leads to the following expression for the mass density

\[
\rho = -\frac{1}{4\pi G} \left( \sigma_u^2 \frac{\partial^2 \ln n}{\partial R^2} + \sigma_w^2 \frac{\partial^2 \ln n}{\partial z^2} + 2D_{uw} \frac{\partial^2 \ln n}{\partial R \partial z} + \left( \frac{\sigma_u^2}{R} + \frac{\partial \sigma_u^2}{\partial R} + \frac{\partial D_{uw}}{\partial z} \right) \frac{\partial \ln n}{\partial R} + \left( \frac{D_{uw}}{R} + \frac{\partial D_{uw}}{\partial R} + \frac{\partial \sigma_u^2}{\partial z} \right) \frac{\partial \ln n}{\partial z} + \frac{\partial^2 \sigma_u^2}{\partial R^2} + \frac{\partial^2 \sigma_w^2}{\partial z^2} + 2 \frac{\partial^2 D_{uw}}{\partial R \partial z} + 2 \left( \frac{\partial \sigma_u^2}{R} \frac{\partial R}{\partial \sigma_u} \right) + \frac{\partial(\nu^2)}{R} + 2 \frac{\partial D_{uw}}{\partial z} \right) .
\]

What should be noted in the final expression given in Eq. (3.14) is firstly that the number density of stars appears only in the shape of derivatives of the logarithm of the number density. This is useful as then not all stars of the Milky Way need to be observed, but a complete sub-sample like for example all Sun-like stars or all supergiants will be enough. This will be exploited in Sect. 4.1. In Sect. 4.2 the fact that velocities appear in mean values and that the expression holds derivatives will be dealt with.

The derived expression for mass density in Eq. (3.14) has not been encountered in the literature during research for this thesis. It is an important equation for this work and is used and discussed in Chapters 4, 5 and 6.
Chapter 4

Method

In this chapter, first Sect. 4.1 describes the creation of a simulated dataset and second Sect. 4.2 explains how the mass density expression derived in Chapter 3, Eq. (3.14), can be used to find the mass density in the galaxy model the data are created in.

4.1 Simulated Gaia data

To test the derived expression of Galactic mass density given in Eq. (3.14), stellar data are needed. As Gaia is in the process of gathering data, the data used in this work are simulated. This is practical as tests of the method can be run on datasets representing very simple galaxy models and the results can be predicted to be correspondingly simple. The idea in this work is to test the mass density estimation method and if the used model is not fully consistent with the Milky Way, the results are therefore almost equally valuable.

This section handles the simulated data and it is quite large, therefore it deserves a summary. First, Sect. 4.1.1 describes the analytic potential model used to represent the gravitational force field of the Galaxy. Section 4.1.2 explains how stellar orbits are integrated in a realistic way in the potential model and in the end of the section it is described how they can constitute the simulated dataset. Finally, in Sect. 4.1.3 the simulated dataset is modified to resemble real Gaia data.

4.1.1 Choice of potential

The potential used to represent the gravitational force field of the Milky Way is the analytic, axisymmetric potential of Paczyński (1990). It consists of a superposition of three components representing the bulge, the disk, and the halo of the Galaxy. The total potential, $\Phi$, can then be written as

$$\Phi = \Phi_b + \Phi_d + \Phi_h,$$

(4.1)

where the subscripts corresponds to the contribution from the bulge, the disk and the halo respectively. As the mass density can be found by differentiating the potential twice as seen in Poisson’s equation, Eq. (3.9), also the mass density is a superposition of contributions from the three components.

The disk and the bulge are both represented by a potential from Miyamoto & Nagai (1975), which is written in the following way

$$\Phi_{MN}(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}},$$

(4.2)

where $M$ corresponds to the total mass of the component and $a$ and $b$ are parameters (dimension length) which define the radial extent ($b$) and degree of flattening ($a/b$). $G$ is the gravitational constant.

The halo is described by a spherical potential of the following form

$$\Phi_h(r) = \frac{GM_c}{r_c} \left[ \frac{1}{2} \ln \left( 1 + \frac{r^2}{r_c^2} \right) + \frac{r_c}{r} \arctan \left( \frac{r}{r_c} \right) \right],$$

(4.3)
where \( r_c \) is the radius at which the density is half of the central density and called the core radius. The mass inside the core radius is denoted \( M_c \). This potential is extending to infinity, which is not the case of the Milky Way halo, but as only a part of the Galaxy will be considered, this fact is not of importance.

For these two types of potentials, Paczyński (1990) fitted the parameters to make the model resemble the Milky Way following observations of the rotation curve (Burton & Gordon 1978; Binney & Tremaine 1987) and the local mass density of \( \rho_0 = 0.18 \, M_\odot \, \text{pc}^{-3} \) (Binney & Tremaine 1987). The parameters of the components where then found to create a best fit for the following values

\[
\begin{align*}
\text{Disk} & : a = 3700 \, \text{pc} & b = 200 \, \text{pc} & M = 8.07 \cdot 10^{10} \, M_\odot \\
\text{Bulge} & : a = 0 \, \text{pc} & b = 277 \, \text{pc} & M = 1.12 \cdot 10^{10} \, M_\odot \\
\text{Halo} & : r_c = 6000 \, \text{pc} & M_c = 5.0 \cdot 10^{10} \, M_\odot 
\end{align*}
\]

As \( a = 0 \, \text{pc} \) for the bulge, its potential reduces to a spherical Plummer model (Plummer 1911; Dinescu et al. 1999; Binney & Tremaine 2008).

In Fig. 4.1 the potential and mass density of the different components and the full model can be seen. The rotation curve of the model is shown in Fig. 2.4 together with the contributions from the different components. In Fig. 4.2 the contributions to the potential and mass density in the midplane \( (z = 0) \) from the different components of the model is shown. It is worth to notice that the main contribution to the mass density in the region Gaia will observe comes from the disk.

More recent estimations for the local mass density than what is used in the model from Paczyński (1990) comes from the predecessor of Gaia, the Hipparcos mission, which concludes that it is lower than before expected. Holmberg & Flynn (2000) estimate the Solar neighbourhood mass density to \( \rho_0 = 0.102 \pm 0.010 \, M_\odot \, \text{pc}^{-3} \). Crézé et al. (1998) find a value of \( \rho_0 = 0.076 \pm 0.015 \, M_\odot \, \text{pc}^{-3} \) and Pham (1997) derive the value \( \rho_0 = 0.09 \pm 0.01 \, M_\odot \, \text{pc}^{-3} \). However, as previously mentioned, the deviation between the model and the current estimate of mass density is irrelevant in this work as the aim is to test the method with which the mass density can be estimated.

### 4.1.2 Orbit integration

In the Galactic plane of the Milky Way, stars are moving in epicyclic orbits, each with different deviation from the circular motion at the specific radius depending on the peculiar velocities of the stars.

To create the simulated dataset, stellar orbits are integrated in the potential model of the Galaxy. With time the orbits adapt to the Galactic potential and can be used to estimate the mass density distribution. The orbits can theoretically be oriented arbitrarily and still be useful to probe the mass density in the model, but in an actual disk galaxy stars are distributed in a specific way (e.g., high number density in the disk). Therefore, to be able to investigate the possible resolution in real data (resolution of the estimated mass density map will be seen to depend on the number of stars through statistics), the stars are in this model distributed in a

\(^1\)Measurements in the Solar neighbourhood. Surface density estimated to \( \Sigma = 75 \, M_\odot \, \text{pc}^{-2} \) between \( z = -700 \, \text{pc} \) and \( z = 700 \, \text{pc} \).
Figure 4.1: Potential (left column) and mass density (right column) for the three components and the superposition of them in the model by Paczyński (1990). From top the rows correspond to: bulge, disk, halo, total. Contours correspond to the values of the ticks in the colour bars.
Figure 4.3: Evolution of the stellar distribution during an integration of about 2 billion years. The spatial coordinates $R$, $\theta$ and $z$ are shown in the left column from top to bottom and the velocity coordinates in the right column with $u$, $v$ and $w$ from top to bottom. The dynamical mechanism phase mixing is visible in all the plots except the azimuth angle $\theta$ as no non-axisymmetric structures are included. The mechanism is visible through the change from the initial distribution to the final stabilised distribution.
realistic way. Spatially, the stars are initially distributed with higher concentration in the inner regions and with an exponential fall-off with a scale length of \( R_L = 4 \) kpc (Binney & Tremaine, 2008). Vertically, the stars are distributed in a normal distribution with a standard deviation of \( \sigma = 300 \) pc over the Galactic plane, while in the azimuth angle the stars are uniformly distributed. Due to the extent of the Galactic disk and the low probability of being able to accurately probe the Galactic bulge with Gaia, stars are only initialised within \( 3 < R < 15 \) kpc. In velocity space, the stars are given two components where the first is the circular velocity and the second a randomly directed peculiar velocity normally distributed with a standard deviation of \( \sigma = 10 \) km/s.

The orbits of the stars are found by numerical integration using the above described initial conditions. The procedure in numerical integration of this kind is that the position and velocity of a star after a small amount of time is calculated by solving the following differential equations

\[
\begin{align*}
\frac{\partial R}{\partial t} &= u \\
\frac{\partial \theta}{\partial t} &= v \\
\frac{\partial z}{\partial t} &= w \\
\frac{\partial u}{\partial t} &= \frac{v^2}{R} - \frac{\partial \Phi}{\partial R} \\
\frac{\partial v}{\partial t} &= -\frac{uv}{R} - \frac{\partial \Phi}{\partial \theta} \\
\frac{\partial w}{\partial t} &= -\frac{\partial \Phi}{\partial z},
\end{align*}
\]  

(4.5)

which derive from the relations between position and velocity, velocity and acceleration and acceleration and potential, see equation Eq. (3.2). The equations are solved for each star many times in a row and in that way the trajectories of the stars are found. In this work, the orbit integration is done in MATLAB using the Runge Kutta integrator ode45, which is of order 4 and 5 and has adaptive time steps. A relative tolerance of \( \text{RelTol} = 10^{-7} \) (i.e., the precision is \( 10^{-5} \)%) has been applied for the energy and angular momentum to remain conserved throughout the integration\(^2\). The relative error after the integration rises typically to max values in the order of \( 10^{-11} \) for angular momentum and \( 10^{-6} \) for energy.

In Fig. 4.3 the evolution of the stellar distribution in the six phase space coordinates is shown. From the figure, it is clear that the dynamical mechanism phase mixing has been active. Phase mixing describes how particles participating in a system move out of phase from each other as their coordinates evolve. Here, for example in the case of the radius \( R \) in the upper left panel of Fig. 4.3, it is seen how the stars' radii slowly mix as the stars assemble at different radii at different times. This happens because the epicycle frequency increases with radius and thus stars moving outwards from an inner radius catch up with stars at radii further out. For the motion in vertical direction, the corresponding period is shorter and as can be seen in the panel for the coordinate \( z \), the distribution adjusts much faster than in the case of \( R \). The radial coordinate \( R \) is seen to be the coordinate which needs longest time to adapt and intake a stable distribution. Because of this, the dataset is assessed as adapted after the threshold time of about 1 500 Myr, when the orbits are ready for use. Between this threshold time and the ending time of the integration, a particular time is randomly picked for each orbit at which the position and velocity of the star is noted in a text file. Why a random time is used and not one specific time for all stellar orbits is simply to avoid minor remaining structures. The text file is the simulated Gaia data and it contains in total 11 050 438 stars out of which 7 224 021 \(^3\) are within the region of the Galaxy which will be treated in the future sections (\( 4.5 < R < 12 \) kpc and \( |z| < 1 \) kpc). All stars cannot be used as they will seem to move in and out from the limits when measurement errors and extinction are added in the next section. Also, as is seen in Fig. 4.4 stars need to be initialised outside the limits because they will move in and out during their orbits.

\(^2\)Energy: \( E = \frac{v^2}{2} + \Phi(r) \), conserved component of angular momentum: \( L_z = Rv \).

\(^3\)This number is only true for one dataset, as in the others measurement errors are added which make the stars seem to be in a different place than they actually are. Also, as will be explained in Sect. 4.2 a trick is used to double the dataset. This by mirroring the data over the disk, which is validated by the symmetry over the Galactic disk in the potential model.
Chapter 4: Method

Figure 4.4: Result from 10,000 stars integrated from $R = 3$ kpc and 10,000 stars integrated from $R = 15$ kpc. To avoid bias, the dataset used lies within $4.5 < R < 12$ kpc as this region is not reached by stars moving in and out of the earlier limits. Further out, the stars spread more and therefore is the edge cut more on the outer edge.

4.1.3 Simulating reality

The simulated Gaia data described in the previous section consists of data observed with perfect precision and also spans all around the Galactic centre (all values of the azimuth angle are allowed). This is not what will be achieved by Gaia, firstly because Gaia will have uncertainties in the measurements, secondly because the Galactic bulge will obscure large parts of the opposite side of the disk and thirdly because the magnitudes of the stars will limit the visible amount. The stellar magnitudes in turn depend on distance and the obscuration due to Galactic dust, an effect called extinction. This section will treat these effects and modify the simulated Gaia data to take them into account and make the data more realistic.

4.1.3.1 Data within $r = 10$ kpc

To firstly cut off the data beyond a visible sphere with radius $r = 10$ kpc (Heliocentric, spherical coordinates), has its reasons mainly in the parallax (and thus the distance) uncertainty being too large. This is seen in Fig. 2.3, where the relative parallax uncertainty is the smallest 10%. By doing this already on the perfect dataset is useful for comparison with the next two datasets as roughly the same amount of stars will be included and thus the statistical support will be approximately the same.

4.1.3.2 Measurement errors

Ultimate tracer star Before adding measurement errors to the dataset, a stellar tracer star should be chosen so that the errors can be customised to this specific spectral type of star. As is seen in Fig. 2.3, the brighter and more massive main sequence stars together with the giant stars seem to have the smallest measurement uncertainties. As the measurement uncertainties correspond to the standard deviation (dispersion) in what here is set as a Gaussian shaped error distribution, a small measurement uncertainty also would generate generally smaller measurement errors.

Before choosing the spectral type performing best in Fig. 2.3, it is important to verify that this type of star is common enough to constitute a good statistical sample also in reality. Additionally, ultimately, the tracer star should be present in every little nook of the Galaxy, so that it can be used to probe the matter distribution everywhere. Therefore, in the following, the distribution and amount of visible stars of several different spectral types are investigated using the so-called Besançon model. This model is applied as it uses realistic distributions of different spectral types of stars.

The Besançon model The Besançon model [Robin et al., 2003] was created at the Observatory of Besançon, France, and is a model of the stellar populations in the Milky Way, which in the considered Galactic model
has four components; a thin and a thick disk, a spheroidal halo and a bulge. Thus, it has no non-axisymmetric structures such as spiral arms or a bar even though such components are known to be present in the Milky Way (as described in Sect. 2.2). The Galactic components are each represented by a stellar population provided with a history of the star formation rate (SFR), initial mass function (IMF), age distribution, evolutionary tracks, kinematic properties, metallicity and a population of white dwarfs apart from the populations of main sequence and giant stars. Extinction\(^\text{a}\) can also be accounted for when using the model, then either in a smooth way or by distributing clouds.

In the investigation of the number density of visible stars, the regions included corresponded to different vertical directions above the Galactic disk and in different horizontal directions compared to the Galactic centre. A smooth extinction of the customary \(A_V = 0.7\) mag/kpc was applied and the considered stars ranged from O0V to M9V in the main sequence and from G0III to M9III in the giant branch. As only a rough estimation was needed, the applied magnitude limits were in the Johnson \(V\) (\(6 < V < 16\) mag) instead of using the actual limits of the Gaia \(G/G_{BVS}\) (\(G > 6\) mag, \(G_{BVS} < 16\) mag). This because if the Gaia magnitude limits would be used, a more memory costly output would have to be used and the similarities between the magnitudes \(V\) and \(G/G_{BVS}\) are relatively big. A more detailed description of the investigation can be found in Appendix C.

From the investigation with the Besançon model it was found that all spectral types of stars are concentrated in the disk of the Galaxy and therefore will this aspect not affect the choice of tracer star.

For Gaia, the magnitude will limit the visibility for different spectral types of stars as distance increases. The limiting distance is found through the correlation between absolute and apparent magnitude given in Eq. (2.7). In Table 4.1, estimates are given for the minimum and maximum distances within which the different spectral types are visible following the magnitude limits (extinction is neglected). When applying the magnitude limits of \(G > 6\) mag and \(G_{BVS} < 16\) mag to Eqs. (2.7), (2.1) and (2.2) the minimum and maximum distances can be found.

Table 4.1 presents also the absolute magnitude and colour index for the first star in the range given at each row. As a good statistical sample is needed (which means a couple of million stars in total, as will be seen in Chapter 5), it is clear from Table 4.1 that the bright main sequence stars cannot be used as spectral tracer stars for the estimation of Galactic mass density as they are far too few.

**Chosen tracer star** From Table 4.1 and Fig. 2.3 it can be concluded that stars which are numerous and have small uncertainties in the measurements are the range of giants from the K0III to the M4III stars. From the optically thin distance limits shown in Table 4.1 it is clear that the M-giants are most advantageous to use. The M-giants are variable stars\(^b\) (Bányai et al., 2013), which means that they can be difficult to accurately measure, but as a rough estimation is made here they are considered anyway. They are disk stars and are rare above just a few kpc. As many not well understood mechanisms appear in the disk, for example the spiral arms, disk stars are verified as reasonably good tracer stars as they at least can probe these regions well. The tracer star chosen in this work is therefore the M0III stars. Concerning the number of stars needed, the whole range from M0III to M4III would be needed, but as their values are similar, they are all here approximated to be M0III stars.

From the large amount of observable main sequence stars, the possibility arises to probe the mass density in the Solar neighbourhood with higher resolution than in the more extended region probed by the giant stars.

**How to add measurement errors** After determining M0III stars to be the most appropriate tracer stars, the next step is to actually apply measurement errors to the simulated Gaia data. This begins with assigning the colour (\(V - I_C\)) and absolute magnitude (\(M_V\)) of the stellar tracer star, to each star included in the text file containing all data. The individual apparent magnitudes are then calculated through Eqs. (2.1), (2.2) and (2.7). Initially, the extinction, \(A_X\), in Eq. (2.7) is ignored. Dispersions for the normally distributed measurement uncertainties in parallax, proper motion and radial velocity can be found for each star from Eqs. (2.4), (2.5) and (2.6) by providing the colour indices and apparent magnitudes together with the constants \(a\) and \(b\) (see Appendix B). Considering a parameter at a time, an error for each star is generated by randomly drawing a value from a normal distribution with the found dispersion. These errors are then added to the original, perfect values of parallax, proper motion and radial velocity.

The position measurement error is in the order of \(\mu\) as for Gaia (Lindegren, 2005) and this is negligible compared to the size of the grid used (and explained further in Sect. 4.2.1) with bins of smallest angular size of \(10^7\) \(\mu\) as.

\(^a\)see Sect. 4.1.3.3

\(^b\)In variable stars, the emitted flux varies with time.
Chapter 4: Method

Table 4.1: Column 1 shows spectral types included in investigation (magnitudes, colour index and distances correspond to the first spectral type, while the range correspond to the investigation of number of stars). Column 4 and 5 show the inner and outer boundaries within which the star (the type first in the range) would be visible (\( G > 6 \) mag and \( G_{RVS} < 16 \) mag) if no extinction existed. To find these distances the absolute magnitude and colour given in columns 2 and 3 are necessary, see Eqs. (2.1) and (2.2). Column 6 and 7 show the estimated amount of stars within the whole range given in column 1 visible by Gaia. This estimation is done with the Besançon model (Robin et al., 2003).

<table>
<thead>
<tr>
<th>Spectral type</th>
<th>( M_V ) [mag]</th>
<th>( V - I_C ) [mag]</th>
<th>( d_{\text{min}} ) [pc]</th>
<th>( d_{\text{max}} ) [pc]</th>
<th>Number of stars ( 6 &lt; V &lt; 16 ) mag</th>
<th>Number of stars ( 6 &lt; V &lt; 20 ) mag</th>
</tr>
</thead>
<tbody>
<tr>
<td>O0V (– O4V)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>O5V (– O9V)</td>
<td>–5.7(^a)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>2 535</td>
<td>2 674</td>
</tr>
<tr>
<td>B0V (– B4V)</td>
<td>–4.0(^a) –0.31(^d)</td>
<td>1 006</td>
<td>84 538</td>
<td>78 906</td>
<td>171 321</td>
<td></td>
</tr>
<tr>
<td>B5V (– B9V)</td>
<td>–1.2(^a) –0.15(^b)</td>
<td>277</td>
<td>25 469</td>
<td>663 817</td>
<td>2 733 627</td>
<td></td>
</tr>
<tr>
<td>A0V (– A4V)</td>
<td>0.65(^a) –0.02(^b)</td>
<td>119</td>
<td>11 683</td>
<td>1 139 929</td>
<td>6 062 808</td>
<td></td>
</tr>
<tr>
<td>A5V (– A9V)</td>
<td>1.95(^a) 0.16(^b)</td>
<td>66</td>
<td>7 095</td>
<td>1 382 666</td>
<td>10 118 281</td>
<td></td>
</tr>
<tr>
<td>F0V (– F4V)</td>
<td>2.7(^a) 0.36(^b)</td>
<td>47</td>
<td>5 610</td>
<td>2 205 519</td>
<td>23 722 261</td>
<td></td>
</tr>
<tr>
<td>F5V (– F9V)</td>
<td>3.5(^a) 0.51(^b)</td>
<td>33</td>
<td>4 214</td>
<td>10 261 912</td>
<td>252 509 246</td>
<td></td>
</tr>
<tr>
<td>G0V (– G4V)</td>
<td>4.4(^a) 0.65(^b)</td>
<td>22</td>
<td>3 006</td>
<td>11 326 709</td>
<td>219 426 936</td>
<td></td>
</tr>
<tr>
<td>G5V (– G9V)</td>
<td>5.1(^a) 0.73(^b)</td>
<td>16</td>
<td>2 274</td>
<td>2 690 705</td>
<td>123 504 021</td>
<td></td>
</tr>
<tr>
<td>K0V (– K4V)</td>
<td>5.9(^a) 0.86(^b)</td>
<td>12</td>
<td>1 689</td>
<td>4 496 269</td>
<td>120 467 001</td>
<td></td>
</tr>
<tr>
<td>K5V (– K9V)</td>
<td>7.35(^a) 1.22(^b)</td>
<td>6</td>
<td>1 051</td>
<td>1 282 940</td>
<td>55 492 308</td>
<td></td>
</tr>
<tr>
<td>M0V (– M4V)</td>
<td>8.8(^a) 1.78(^b)</td>
<td>4</td>
<td>725</td>
<td>429 612</td>
<td>33 893 820</td>
<td></td>
</tr>
<tr>
<td>M5V (– M9V)</td>
<td>12.3(^a) 2.80(^b)</td>
<td>1</td>
<td>244</td>
<td>1 019</td>
<td>1 322 672</td>
<td></td>
</tr>
<tr>
<td>G0III (– G4III)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1 055 162</td>
<td>6 195 808</td>
</tr>
<tr>
<td>G5III (– G9III)</td>
<td>0.9(^a) 0.91(^b)</td>
<td>117</td>
<td>17 349</td>
<td>6 546 108</td>
<td>63 835 654</td>
<td></td>
</tr>
<tr>
<td>K0III (– K4III)</td>
<td>0.7(^a) 1.02(^b)</td>
<td>131</td>
<td>20 185</td>
<td>21 389 536</td>
<td>199 243 193</td>
<td></td>
</tr>
<tr>
<td>K5III (– K9III)</td>
<td>–0.2(^a) 1.70(^b)</td>
<td>230</td>
<td>43 868</td>
<td>4 193 113</td>
<td>12 465 762</td>
<td></td>
</tr>
<tr>
<td>M0III (– M4III)</td>
<td>–0.4(^a) 1.74(^c)</td>
<td>255</td>
<td>49 121</td>
<td>1 767 189</td>
<td>4 249 036</td>
<td></td>
</tr>
<tr>
<td>M5III (– M9III)</td>
<td>–0.3(^a) 3.28(^c)</td>
<td>409</td>
<td>102 715</td>
<td>348 507</td>
<td>1 793 706</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>71 261 143(^*)</td>
<td>1 137 210 134</td>
</tr>
</tbody>
</table>

\(^{a}\) Cox (2000)  
\(^{b}\) Straizys (1992)  
\(^{c}\) Calculated from Johnson’s system given in Cox (2000) with relations from Bessell (1983)  
\(^{d}\) Gaia Science Performance web page, ESA.

\(^{*}\) This number deviates from the number found by Robin et al. (2012) of roughly 150 million stars. The reason can be the approximate magnitude limits used in this investigation.

\[^{\text{http://www.cosmos.esa.int/web/gaia}}\]
4.1.3.3 Extinction

What is extinction? Extinction is the term used to signify decrease in visibility and thus increase in apparent magnitude. Responsible for the extinction in the Milky Way is dust obscuring the light which propagates through the Galaxy. The dust is coupled to gas as the dust particles are of small size and the dust is mostly created from supernova explosions of massive stars, which die so early that they still reside in their nursery of Galactic gas. The Galactic gas and dust appear in the shape of clouds in the Milky Way and thus the structure of extinction is very patchy. When observing from the Solar system, there are therefore some directions which are sparsely occupied with clouds, also called windows, in which the extinction is low and very distant stars can be observed. On the other hand are there also other directions which might be more promising for observing clouds where the extinction is high and only nearby stars can be seen. An example of a window of low extinction is Baade’s window which is a $\sim 40 \times 40$ arcmin$^2$ region with extinction varying within $1.26 < A_V < 2.79$ mag and directed towards the Galactic centre (Stanek, 1996).

For many years, attempts to map the Galactic extinction have been made. Early on, for example Burstein & Heiles (1978) constructed a dust map by deriving the dust column density from the assumed connection to the observed abundance of neutral hydrogen, HI. A more accurate mapping was made by Schlegel et al. (1998) who used the microwave data from the IRAS satellite and the DIRBE instrument at the COBE satellite. From the microwave data, dust temperatures could be derived and from these the dust column densities inferred. An even newer map was provided by Schlafly & Finkbeiner (2011) who re-calibrated the dust extinction found in Schlegel et al. (1998) by also incorporating the data from the Sloan Digital Sky Survey (SDSS).

Another effect from the dust is the interstellar reddening. It is a phenomenon occurring because the dust scatters away short wavelengths leaving the longer redder wavelengths emerging from the cloud. Therefore, in regions with high extinction, the emerging starlight also looks redder than expected.

Model for extinction In the Gaia mission, extinction will damage the visibility of stars in the Galaxy. To account for such a change in the observation of stars in the simulated Gaia data, a model for extinction is used here to calculate the amount of extinction for each of the stars in the data file.

The model comes from Amôres & Lépine (2005) and does not take into account the patchy structure of the Galactic extinction, but instead uses a smooth distribution. A smooth extinction map is not realistic but gives an idea of how much damage extinction can cause. However, it should be kept in mind that in reality extinction is patchy and thus data may be more or less affected than what this model shows.

The model is based on observations of the distribution of neutral (HI) and molecular (H$_2$) hydrogen (H$_2$ through observing the molecule CO) and assumes that the dust is well-mixed with the gas. From the hydrogen an extinction in the Johnson $V$ band can be found and affects thus the observations of the stars as in Eq. (2.7), but now for the Johnson $V$ band so that the equation writes

$$ V = M_V + 5(\log_{10} d - 1) + A_V. \tag{4.6} $$

An interpretation of the extinction model of Amôres & Lépine (2005) can be found in Appendix [D] and Fig. 4.5 shows how the extinction model looks like.

Effect on stellar sample As extinction is an increase in magnitude, it then adds a term to the equation for apparent magnitude, as is seen in Eqs. (2.7) and (4.6). The interstellar reddening is an effect where the dust scatter away shorter wavelengths and cause the colours of the stars to change as follows

$$ (V - I_C)_{\text{reddened}} = (V - I_C)_{\text{intrinsic}} + E_{V - I_C}, \tag{4.7} $$

where the colour excess $E_{V - I_C}$ is found by Tammann et al. (2003) to be related to the other colour excess $E_{B - V}$ through

$$ E_{V - I_C} = (1.283 \pm 0.011) \cdot E_{B - V}, \tag{4.8} $$

by fitting to Cepheids. The colour excess is related to the extinction $A_V$ through the ratio of total to selective extinction, $R_{B - V}$, in the following way

$$ E_{B - V} = \frac{A_V}{R_{B - V}}. \tag{4.9} $$
Figure 4.5: Extinction of different regions in the model of Amores & Lépine (2005), where the Sun is located at $x = 8$ kpc. The region of $R < 1.2$ kpc is not included in the model and is therefore obscured in these plots. Central panel is in the Galactic plane $z = 0$, bottom panel is a cut through the Galaxy at $y = 0$ and the right panel is a cut at $x = 8$ kpc. (On the far left part of the central panel the extinction seem to unphysically decrease along the line of sight. This is an effect coming from the model, see Eq. (D.3), and is further explained in Appendix D. As is seen in Fig. 4.6 the part of the Galaxy included in the mass density estimation is not affected by this decrease with the line of sight.)
Figure 4.6: Discrete map over the visibility of M0III stars through G_{RVS} magnitude. Stars with $G_{RVS} > 16$ mag are invisible for Gaia, which means beyond yellow towards red here. The plot shows the Galactic plane, $z = 0$, and the Sun is located at $x = 8$ kpc. The limits of the dataset ($4.5 < R < 12$ kpc) are marked with white and the visible sphere of Gaia used in this work ($r < 10$ kpc) by black.

By combining Eqs. (4.9) and (4.8) the colour excess for the colour index $V - I_C$ can be found to be

$$E_{V-I_C} = (1.283 \pm 0.011) \cdot \frac{A_V}{R_{B-V}}.$$  

The factor $R_{B-V}$ is given by Cox (2000) to be $R_{B-V} = 3.1$ (the value for dilute interstellar medium). In this work, the uncertainty in Eq. (4.8), and thus Eq. (4.10), is neglected when calculating the interstellar reddening for the stars.

The extinction will be equal for all spectral types of stars in the same position, but the difference will be that faint stars will no longer be observable while bright stars will merely receive a larger measurement error. An incomplete sample of stars will damage the estimation of the gradient of the number density of stars which is undesirable as it will affect the final results of the estimation of the mass density. This will come back with the results in Chapter 5. The magnitude limits of the satellite Gaia is $G > 6$ mag and $G_{RVS} < 16$ mag for measurement of radial velocity to be possible. To derive the Gaia magnitudes as in Eqs. (2.1) and (2.2) also the colour is required, which in turn involves the interstellar reddening in the magnitude limits. To check whether the chosen tracer stars will be visible throughout the region to probe, the magnitude of a tracer star at every position is calculated using Eq. (4.6) and compared with the magnitude limits. The result is shown in Fig. 4.6 where the apparent Gaia $G_{RVS}$ magnitude for the M0III stars is plotted.

It can be seen in Fig. 4.6 that not all tracer stars will be possible to probe in the region, and this will cause damage to the results as will be seen in Chapter 5 (in particular, see Fig. 5.4).

4.2 Apply expression to data

In the previous section, it was described how the simulated Gaia data were created. This section explains how the mass density expression derived in Chapter 3, Eq. (3.14), is applied to the created data to estimate the
(a) **Grid 1:** Grid within $4.5 < R < 12$ kpc and $|z| < 1$ kpc. 20 bins in $R$ and 25 (50 when doubled) bins in $z$. Used for the dataset perfect$p_{90}$.

(b) **Grid 2:** Grid within $4.5 < R < 12$ kpc, $|z| < 1$ kpc and $r < 10$ kpc (marked by blue line). 20 bins in $R$ and 25 (50 when doubled) bins in $z$. Used for the datasets $r_{10}p_{90}$, ME$p_{90}$ and MEext$p_{90}$.

(c) **Grid 3:** Grid within $R < 12$ kpc, $x \geq 5.5$ kpc (red line), $r < 10$ kpc (blue line) and $|z| < 1$ kpc. 20 bins in $R$ and 25 (50 when doubled) bins in $z$. Dashed line shows inner limit for the grid in the first grid. Used for datasets $x_{cut5}p_{90}$ and $x_{cut5phom}p_{90}$.

Figure 4.7: Grids used together with the simulation in the axisymmetric potential. Position of Sun $((x, y, z) = (8 \text{ kpc}, 0, 0))$ is marked in the left panels. The shaded parts are not used, but present for comparison.
mass density of the used potential model. The estimation can then be compared to the true mass density by deriving it directly from Poisson’s equation, Eq. (3.9), applied on the potential model given in Sect. 4.1.1.

### 4.2 Grid

The terms in the expression in Eq. (3.14) consist only of observable parameters in shape of position, velocities and number density of stars. What should be noted is that all the terms in the expression holds a mean value of a parameter, e.g., velocity dispersions or number density. This means that a single star cannot be used to estimate the mass density at its location, but a smaller set of stars must be used to find the mean values and from them create an average of the mass density in that region. This is the reason why the created dataset will be divided into bins or in other words, that a grid is constructed. The grid then decides the resolution of the estimated mass density map.

What affects the design of the grid is the amount of stars within every bin as the mean values, and thus the estimation, will be better if the statistical sample is large. The perfect grid would have equal amount of stars in every bin so that the precision of the mass density estimation would not differ between the bins because of a statistical difference. To achieve something close to this perfect case, the dataset is divided into bins in the radial and vertical direction, but with decreasing size towards the Galactic plane and Galactic centre as there are expected to reside a higher concentration of stars. The decrease in bin size is found by the following equations

\[
s_R = \ln(R)
\]

\[
s_z = \text{arcsinh}(z) = \ln(z + \sqrt{z^2 + 1})
\]

for \( R \) and \( z \) respectively. The result of this binning is seen in Fig. 4.7 where the rings in the left panels can be seen to have smaller and smaller intervals towards the Galactic centre and in the right panels also the decrease is seen in the vertical direction. As can be seen, only the binning in the upper part of the Galactic disk is shown in the right panels. This is because the binning in the lower part is perfectly symmetric to the upper part and the reader is asked to be imaginative.

Figure 4.7 shows three grids. Grid 1 is the grid described so far. As it is not realistic that Gaia will be able to probe this entire region, only the sphere within 10 kpc from the Solar system, \( r < 10 \) kpc, is considered in Grid 2. This is marked by a blue circle. By using Grid 3 the regions with too much extinction can be excluded (compare with Fig. 4.6), which is useful as it causes wrongly estimated number density of stars and too large measurement errors. In this grid the region of \( x \gtrsim 5.5 \) kpc is considered, which is a limit marked with a red line.

![Figure 4.8: Left panel: the values for the logarithm of the number density (ln \( n \)) in each bin is plotted in the meridional plane. Right panel: a surface is fitted to the data seen in the left panel. This surface is used to find the derivatives of ln \( n \) in each bin (at each dot).](image)

### 4.2.2 Surface fits

In each of the bins shown in the grids in Fig. 4.7 the velocities of the stars are used to find the elements of the dispersion matrix (the dispersion matrix is presented in Eq. (3.12)) and the stars are counted to find the number density. The position used for the stars in the bin is the middle of the bin.
However, in Eq. (3.14) it is apparent that for finding a mean value of the mass density in every bin, derivatives of the different parameters are needed. This is something which cannot be produced from the single mean values in a bin. To solve this issue an estimation of the terms is done by fitting a surface to the calculated parameters in all bins and then from the surface extract the derivatives. As the surface is smoother than the data, also probably better estimates of the already found parameters can be made. In Fig. 4.8 the method of surface fitting is demonstrated on the logarithm of the number density, ln \( n \). The same technique is used for the other derivatives present in Eq. (3.14).

The method with which the surface fits are produced is called a bicubic B-spline and consists of a superposition of 2D functions adjusted to match the data (from private communication with prof. L. Lindegren). The precision with which the surface can fit the data can be tweaked by several parameters, therefore to perfectly adapt the surface is complex. In this work, moderate amount of focus has been dedicated to tweaking the parameters of the surface fits and therefore, as will be seen in Chapter 5, artefacts from imperfection of the surface fits occur in the results.

Using the estimations of the derivatives of the parameters together with estimations of also the parameters, both from surface fits, the expression for mass density can finally be used on the data. This is done in every bin and therefore depending on the amount of stars available and the design of the grid, the resolution of the result is different.
Chapter 5

Results

5.1 Analysis of mass density expression

In the expression used to find the mass density distribution in the Galaxy model, Eq. (3.14), there have been some terms found to be of particular importance. These are all correlated with the distribution of stars and velocities in the vertical direction. The most important terms are the second, ninth and eleventh terms, that is \( \sigma_w^2 \frac{\partial^2 \ln n}{\partial z^2} \), \( \sigma_w^2 \frac{\partial \sigma_v^2}{\partial z} \frac{\partial \ln n}{\partial z} \) and \( \sigma_w^2 \sigma_v^2 \frac{\partial^2}{\partial z^2} \).

5.2 Presenting mass density estimations

When presenting the results in this chapter, the data created by simulating stellar orbits in the axisymmetric potential model of Paczyński (1990) are divided into different datasets depending on how the data have been treated. For this basic potential model, all steps in the way of making the data more realistic are presented. This includes seeing degeneration of the results by decreasing the amount of stars in the cut out of \( r < 10 \) kpc (dataset perfect\_p90 to \( r10\_p90 \)), the difference in the estimation due to measurement errors (dataset \( r10\_p90 \) to ME\_p90), reach of magnitude limits leading to exclusion of stars and increase of measurement errors, both due to extinction (dataset ME\_p90 to MEext\_p90). The outcome is then improved by cutting off the sections with too large measurement errors and lack of stars and also by including the possible use of photometrically found distances to distant stars (datasets \( xcut5\_p90 \) and \( xcut5phom\_p90 \)).

This information is also given in Table 5.1. For a helpful figure to understand which part of the data is used, see the grids in Fig. 4.7. In the following results, a trick has been used to decrease the amount of computation time needed, and it is that as all potential models are symmetric above and beneath the Galactic disk, the data can be projected on both sides and thus doubled (this double number of stars is what is given in Table 5.1). This trick causes effects in the dataset to occur symmetrically as is seen in the following sections.

The results shown will be displayed visually by colour plots of the mass density estimation, difference and relative difference from the actual mass density and 2D in the Galactic plane and over the Galactic disk at the location of the Solar System. Each figure contains eight panels, which are referred to as panel 1 to 8, with counting left to right and top to bottom.

5.3 Simulations in the Paczyński (1990) potential model

In this section, the results from the simulated stars in the axisymmetric potential model of Paczyński (1990) is presented (see Sect. 4.1.1). In every step of increasing the degree of reality the results are presented. For an overview of the presented datasets, see Table 5.1

5.3.1 perfect\_P90

Figure 5.1 presents the results from the full dataset of 14 448 042 stars, not perturbed by either measurement errors or extinction. The grid used is Grid 1 in Fig. 4.7. As can be seen already in the two uppermost panels of Fig. 5.1, the method manages to recreate the mass density distribution in the model and show roughly the same scale length and scale height in the density profile. Above and beneath the disk, regions of negative density are
Chapter 5: Results

Figure 5.1: Results from the dataset with perfect precision generated in the axisymmetric potential model. From top to bottom and left to right (panel 1 – 8): 1. True mass density, 2. Estimated mass density, 3. Difference between true and estimated mass density, 4. Zoom-in of the difference seen in panel 3., 5. Relative difference between the true and estimated mass density, 6. Zoom-in of the relative difference seen in panel 5., 7. Mass density estimation in the Galactic plane, \( z = 0 \) pc, 8. Mass density estimation over the Galactic disk at the Solar radius, \( R = 8 \) kpc. Dataset has in total 14,448,042 stars.
Table 5.1: Properties of the datasets created from the simulation in the axisymmetric potential.

<table>
<thead>
<tr>
<th>Name</th>
<th>$r &lt; 10$ kpc</th>
<th>M.E.</th>
<th>Ext.</th>
<th>$x \geq 5.5$ kpc</th>
<th>$d_{phom}$</th>
<th>Grid</th>
<th>Nbr of stars</th>
</tr>
</thead>
<tbody>
<tr>
<td>perfect_p90</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>1</td>
<td>14 448 042</td>
</tr>
<tr>
<td>r10_p90</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>2</td>
<td>6 719 776</td>
</tr>
<tr>
<td>ME_p90</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>2</td>
<td>6 899 438</td>
</tr>
<tr>
<td>MEext_p90</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>2</td>
<td>5 566 812</td>
</tr>
<tr>
<td>xcut5_p90</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>3</td>
<td>2 623 666</td>
</tr>
<tr>
<td>xcut5_phom_p90</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>3</td>
<td>2 447 252</td>
</tr>
</tbody>
</table>

Notes. $r < 10$ kpc corresponds to using only the data within the sphere with that radius. M.E. signifies whether measurement errors have been added or not. Ext. refers to extinction added or not. $x \geq 5.5$ kpc refers to the cut in the grid at $x = 5.5$ kpc which is the difference between Grid 2 and Grid 3. Grid gives which grid has been used (see Fig. 4.7). Nbr of stars is the column where the total number of stars is given (after doubling the dataset by mirroring over the Galactic plane).

estimated to be located. This is not physical and comes from the surface fit described in Sect. 4.2 not being perfect when fitting the logarithm of the number density ($\ln n$). The fit could be made better if the weighting in the fit would be perfect.

In the next-coming four panels in Fig. 5.1, in the leftmost panels the difference between the true and the estimated mass density is given first in $M_\odot$ pc$^{-3}$ and then in the relative difference. The rightmost panels are zoom-ins of the disk of the corresponding left panel. In these, the colour bar could be narrowed as the estimation is better in the disk.

In the lowest two panels, the estimation is compared to the true mass density in first the Galactic plane ($z = 0$ pc) and then at the Solar radius ($R = 8$ kpc). As can be seen, the deviation between the estimated and the true mass density is small everywhere except where the surface fit mentioned above spoils the estimation above and beneath the disk.

The statistical sample of stars (about 14 million) seem to be enough for the reconstruction of the model’s mass density using the described method configuration.

5.3.2 $r10\_P90$

Figure 5.2 show the results from the perfect dataset cut out from the sphere of $r < 10$ kpc around the Sun (Grid 2 in Fig. 4.7). Comparing the mass density estimation in this case with the estimation found in Fig. 5.1 indicates that even though the amount of stars included, and thus the statistical sample, has decreased roughly by half, the method still manages to reconstruct the mass density distribution to large extent. Now, additional blue streaks appear above and beneath the disk, which firstly are symmetric due to the fact that the stars have been mirrored over the disk, and secondly also are due to the surface fitting not being perfect. As the same stars are used also in Fig. 5.1 the difference that more stars are being included there, indicates that the surface fitting also gets better from a larger statistical sample.

The next-coming four panels are again difference and relative difference between the true and estimated mass density. Comparing with Fig. 5.1 it is seen that more intense blue and red spots have appeared, which indicates a slightly worse mass density estimation.

However, the lowest two panels showing the density in the Galactic plane and at the Solar radius, still agrees well with the true mass density in the region of the disk.

5.3.3 ME\_P90

After adding measurement errors to the data, the stars included in the grid used (see Grid 2, Fig. 4.7) are not the same as in the case described in Sect. 5.3.2. This is because when the position measurement is uncertain enough, some stars may seem to be somewhere they actually are not and can either be included or excluded from the investigation. In this case the number of stars increases from about 6.7 to roughly 6.9 million stars.

However, the mass density estimation remains similar to the true mass density as is seen in Fig. 5.3. Compared to Fig. 5.2 the blue streaks above and beneath the disk are now more prominent and also the innermost
Figure 5.2: Results from the dataset with perfect precision, but which is limited by the sphere \( r < 10 \) kpc centred on the Solar system. From top to bottom and left to right (panel 1 – 8): 1. True mass density, 2. Estimated mass density, 3. Difference between true and estimated mass density, 4. Zoom-in of the difference seen in panel 3., 5. Relative difference between the true and estimated mass density, 6. Zoom-in of the relative difference seen in panel 5., 7. Mass density estimation in the Galactic plane, \( z = 0 \) pc, 8. Mass density estimation over the Galactic disk at the Solar radius, \( R = 8 \) kpc. Dataset has in total 6 719 776 stars.
Figure 5.3: Results for the dataset where measurement errors generated for M0III stars have been added to the dataset perfect\_p90 and then the sphere $r < 10$ kpc has been cut out. From top to bottom and left to right (panel 1 – 8): 1. True mass density, 2. Estimated mass density, 3. Difference between true and estimated mass density, 4. Zoom-in of the difference seen in panel 3., 5. Relative difference between the true and estimated mass density, 6. Zoom-in of the relative difference seen in panel 5., 7. Mass density estimation in the Galactic plane, $z = 0$ pc, 8. Mass density estimation over the Galactic disk at the Solar radius, $R = 8$ kpc. Dataset has in total 6 899 438 stars.
Chapter 5: Results

region of the disk included is worse estimated. Both these effects are artefacts of the measurement errors damaging the dataset.

As is seen in the coming four panels showing the difference and relative difference the estimation is worse than in the two previous cases seen in Figs. 5.1 and 5.2. It is here even clearer that the central density is underestimated from the purple regions in the zoom-in of the difference between the true and the estimated mass density.

In the two last plots it is seen that the mass density estimation still manages to reconstruct the environment in the Galactic disk and over the disk at the Solar radius in a good way.

5.3.4 MEext_P90

The results shown in Fig. 5.4 belong to the dataset when first extinction and then measurement errors have been added to the data, and after that the stars beyond $r = 10$ kpc have been excluded. Due to the extinction the magnitude limits exclude stars in the central bins as is seen in Fig. 4.6. In other bins where the extinction is large, the increased magnitude and reddening causes huge damage to the data through the measurement errors. These effects are clearly visible in the bad mass density estimation made with the same grid as before (see Grid 2, Fig. 4.7) visible in the top right panel of Fig. 5.4. In this dataset the amount of participating stars have decreased from the previous 6.9 million to about 5.6 million.

In the difference and relative difference plots it is visible that the largest error is found in the central regions. This might have been suspected from the top right panel of Fig. 5.4. Note that the colour bars had to be extended to higher numbers compared to before, so the situation is graver than what it might look like.

Also the lowest two panels show bad agreement of the estimation with the true mass density in both the Galactic plane and at the Solar radius.

As can be seen in the majority of the plots in Fig. 5.4 the central regions show negative mass density. This is, as mentioned earlier, also the case in some regions above and beneath the disk. Above and beneath the disk, the bad fit comes from not perfectly tweaked parameters together with lower amount of stars (statistical sample is smaller). In the central regions in this case with extinction added, the negative density is again an effect of the surface fits not matching the data completely.

5.3.5 xcut5_P90

As the biggest problem in the mass density estimation when extinction is applied mainly derives from the inner regions of the dataset, a part of the dataset is cut off. This is at $x \approx 5.5$ kpc and leads to Grid 3 in Fig. 4.7 where the shaded region to the left of the red line is excluded. By excluding this region, not only the magnitude limit seen in Fig. 4.6 is avoided, but also the stars with high measurement errors residing in these inner regions are. The amount of stars in the investigation decreases from the 5.6 million in the previous section to 2.6 million due to the cut-off.

Figure 5.5 shows the results for the dataset with extinction and measurement errors. Note now that despite the similar size of the plots, the $x$-axis is actually covering a shorter interval of $R$, now beginning at $R \sim 5.5$ kpc. As is seen in the grid, not all bins cover large regions of space and the result is that the amount of stars in the inner bins decrease. This fact makes the mass density estimation more uncertain in the inner regions. However, what creates the overestimation of mass density in these regions, which is seen in the upper right panel of Fig. 5.5 comes from the still comparatively much higher measurement errors caused by extinction. What should be noted though, is that comparing this plot with the corresponding plot in the previous section, the result is significantly more resemblant to the true mass density distribution shown in the top left panel.

From the following four plots showing the difference and relative difference between the true and estimated mass density, the overestimation is also clearly visible. The difference from the corresponding plots in Fig. 5.4 is that now the estimation is much closer to the true mass density (colour bar changes). Some difference lingers though.

When it comes the the lower two plots showing the mass density estimation in the plane of the Galactic disk and over the disk at the Solar radius, it is visible that the overestimation in the central regions exists. Also, on each side of the disk at some hundred pc appears a slight overestimation, while the actual middle of the disk is underestimated at the Solar radius.
Figure 5.4: Results from the dataset where first extinction was added to the dataset perfect_ext_p90 and with these values measurement errors for M0III stars were generated and added to the data. Finally, the sphere $r < 10$ kpc was cut out. From top to bottom and left to right (panel 1 – 8): 1. True mass density, 2. Estimated mass density, 3. Difference between true and estimated mass density, 4. Zoom-in of the difference seen in panel 3., 5. Relative difference between the true and estimated mass density, 6. Zoom-in of the relative difference seen in panel 5., 7. Mass density estimation in the Galactic plane, $z = 0$ pc, 8. Mass density estimation over the Galactic disk at the Solar radius, $R = 8$ kpc. Dataset has in total 5 566 812 stars.
Figure 5.5: Results from the same dataset as in Fig. 5.4 but where the grid has been restricted by \( x \gtrsim 5.5 \) kpc and thus the bins with wrongly estimated number density have been removed. From top to bottom and left to right (panel 1 – 8): 1. True mass density, 2. Estimated mass density, 3. Difference between true and estimated mass density, 4. Zoom-in of the difference seen in panel 3., 5. Relative difference between the true and estimated mass density, 6. Zoom-in of the relative difference seen in panel 5., 7. Mass density estimation in the Galactic plane, \( z = 0 \) pc, 8. Mass density estimation over the Galactic disk at the Solar radius, \( R = 8 \) kpc. Dataset has in total \( 2.623 \times 10^6 \) stars.
Figure 5.6: Results for a very similar dataset as shown in Figs. 5.4 and 5.5, but where it has been assumed that photometry can sometimes be used to estimate the distance. Same grid used as in Fig. 5.5. From top to bottom and left to right (panel 1 – 8): 1. True mass density, 2. Estimated mass density, 3. Difference between true and estimated mass density, 4. Zoom-in of the difference seen in panel 3., 5. Relative difference between the true and estimated mass density, 6. Zoom-in of the relative difference seen in panel 5., 7. Mass density estimation in the Galactic plane, $z = 0 \text{ pc}$, 8. Mass density estimation over the Galactic disk at the Solar radius, $R = 8 \text{ kpc}$. Dataset has in total 2 447 252 stars.
Chapter 5: Results

5.3.6 \textit{xcut5phom}\textsubscript{P90}

Figure 5.6 shows results from a similar dataset as in Sect. 5.3.5 with the difference that here the distances to stars with large relative parallax measurement uncertainty have been decided through the method of photometric distance determination described in Sect. 2.1.5. As can be seen in the top right panel of Fig. 5.6 and comparing to the corresponding panel of Fig. 5.5, the photometrically determined distances improve the mass density estimation. In the central regions the mass density agrees better to the true mass density shown in the top left panel. The number of stars changes slightly from the fact that the measurement errors decrease in the parallax and the amount of stars participating in this case is 2.4 million.

In the following four plots corresponding to the difference and relative difference between the true and the estimated mass density the colour bars are back to the scale they had initially in Figs. 5.1, 5.2 and 5.3. This means that the estimation has improved compared to the previous steps in Figs. 5.4 and 5.5. Even though the zoom-in plots are quite colourful, what should be noted is that the region around the Sun is very green, corresponding to good agreement with the true mass density and thus possibility to probe this region more accurately. In Fig. 5.7 the relative error in the region 6 \textless R \textless 9 kpc and -50 \textless z \textless 50 pc is shown and as can be seen in this extra zoom-in, the error is less than five percent in the whole region. The error in the Solar neighbourhood is around one percent.

The two lowest panels in Fig. 5.6 show the mass density estimation compared to the true mass density in the Galactic plane and over the Galactic disk. The estimation fits well the true mass density in the inner Galactic plane, where the small deviation in the outer part probably derives from the smaller amount of stars available in outer regions of the model. A similar shape of the estimation over the disk is seen as in Fig. 5.5 but here the actual middle of the disk is better estimated.

This is the most realistic and most accurate mass density estimation created in this work. The used grid and the technique with using also photometrically determined distances can be applied to other datasets of stars simulated in other, more complicated models, such as the one presented in Appendix E.
Chapter 6

Discussion & conclusions

Reconstructing the large-scale structure of the Milky Way  From the simulations in the potential model by Paczyński (1990), described in Sects. 4.1 and 5.3, it can be seen in the resulting estimation in Fig. 5.6 that it will be possible to probe the large-scale structure of the Milky Way with Gaia, using the method described. This, providing the assumptions based on the current knowledge of our Galaxy and Gaia are valid.

In the zoom-in of the relative difference between the true and estimated mass density distribution, panel 6 in Fig. 5.6, it can be seen that within $|z| \lesssim 200$ pc and $x \gtrsim 5.5$ kpc, the estimation error does not increase over 35%. Within the smaller region limited by $6 \lesssim R \lesssim 9$ kpc, $|z| \lesssim 50$ pc and $x \gtrsim 5.5$ kpc, the relative error is less than 5% (see Fig. 5.7). Due to the larger amount of stars present in this region, it might very well be that a finer grid can be introduced there and further resolve details without the precision decreasing much. It will be possible to determine the local mass density with about ten times higher accuracy than the value from the Hipparcos data, as the error in the Solar neighbourhood is found to be about one percent (see Fig. 5.7).

By comparing the results from the datasets with and without measurement errors and extinction (datasets $r_{10, p90}$, $ME_{p90}$ and $ME_{ext, p90}$, see Figs. 5.2, 5.3 and 5.4 and Table 5.1), it can be concluded that extinction, rather than measurement errors, is the main obstacle for achieving accurate density estimates in the considered region. From comparing datasets perfect, $p90$ seen in Fig. 5.1 with $r_{10, p90}$ seen in Fig. 5.2, it is seen that the amount of stars included in the estimation is a relevant factor as perfect $p90$ has roughly twice as many stars as $r_{10, p90}$. The conclusion is that in order to achieve a reasonable estimate the number of stars needed to include is in the order of $1 - 10$ million at least.

A lack of error bar estimates in Chapter 5 can be noted and this is because this investigation is based on the single simulation described in Table 5.1 and presented in Sect. 5.3. Many individual simulations, created in the same way as the presented simulation, could be used to estimate the distribution of mass density estimations (this is a so-called Monte Carlo method). Error bars could then be extracted by comparing all estimated mass densities with each other. Such an investigation has not been carried out, but from a smaller set of similar datasets created during the testing and development of the method, it has been found that the difference between the different datasets is very small. It should also be mentioned that when generating normally distributed measurement errors for each star, these were not saved, but regenerated every time the code was run. This indicates that the differences between the estimated and true mass density visible in all datasets as seen in Figs. 5.1, 5.2, 5.3, 5.4, 5.5 and 5.6 rather are due other effects such as insufficient statistical support, i.e., not enough stars available, or that the surface fitting method used for estimation of derivatives in Eq. (3.14) and explained in Sect. 4.2.2 is not accurate enough.

As mentioned in Sect. 4.2.2, the method with which the derivatives in the mass density expression, Eq. (3.14), are estimated builds on surface fits of the measured data in the meridional plane. The surfaces are constituted of a set of two-dimensional spline functions and to fit each surface in a good way require therefore tweaking of parameters such as smoothness and weighting. Because the imperfection in the choice of these parameters, effects appear in the final estimation. For example, is a region higher above and beneath the Galactic disk estimated to have (un-physically enough) negative density as is seen in the figures in Sect. 5.2. Carrying out the search for perfect parameters was assessed as too time-consuming, and as the current values give reasonable estimations, the progression of the investigation was prioritised. To reach perfection in the surface fits could be very rewarding as then possibly less stars could be used to reach the same accuracy and also regions more sparsely populated a bit above the Galactic disk might be possible to probe. This would be interesting as the contribution of dark matter in these regions is expected to be much higher than in the disk.

41
Chapter 6: Discussion & conclusions

The method is based on statistics through mean values of the observable parameters position, velocities and number density seen in Eq. (5.14) and thus depends on the amount of stars included. With less stars, these values will statistically deviate more from the actual values and perturb the mass density estimation. As seen in Table 4.1 it is not probable that Gaia will observe more than a few million of the chosen tracer stars (M-giants) and this limits the goodness of the mass density estimation. A work-around to avoid this limitation could be to also include other spectral types as tracer stars, but these should be chosen with care as the measurement errors depend strongly on spectral type. What would be interesting to do is to pick a more common spectral type as tracer star and investigate the accuracy with which this type could probe the region closer around the Sun.

Connected to the weighting of the surface fits and the available statistical support is not only the number of stars, but the number of stars present in each bin in the chosen grid (for grid used in this work see Fig. 4.7). Optimally, the grid should have been customised to the true matter distribution in the potential model in such a way that the predicted amount of stars in all bins would be equal. Such a grid would imply that no weighting is needed as the statistical sample in every bin is of equal statistical importance. However, such a grid would induce a better estimation than realistic as the actual matter distribution of the Milky Way is not known. To achieve such a grid, I propose an adaptive algorithm which after estimating the matter distribution once, the grid is recreated following the results achieved and then the matter distribution would be estimated once again. Time restrictions in the project plan did not allow for the application of this type of grid.

Concerning the correlation between the simulated data and the actual stars measured by Gaia, the difference is significant. First, the potential model of Paczyński (1990) is axisymmetric, symmetric over the Galactic plane and has a smooth structure. The Milky Way is a disk galaxy and has thus some reasonable symmetry over the Galactic plane, but axisymmetry does not apply as there are both a central bar and spiral arms present. Also, as stars form in clusters and other dynamical mechanisms affect the Galaxy, its structure is not as smooth as in the model used here. Secondly, the mass density the model assumes in the Solar neighbourhood is predicted already to be inaccurate with almost a factor of two as mentioned in Sect. 4.1. Yet another fact already mentioned in Sect. 4.1.3.3 is that the interstellar dust in the Milky Way is irregularly distributed and causes the extinction to be patchy. This is not included in the extinction model of Amores & Lépine (2005) where the extinction instead is assumed to be smoothly distributed.

Despite the above mentioned differences between the generated data and the stars in the real Milky Way, the investigation carried out is useful as it demonstrates that the direct application of Jeans equations on astrometric data of stars can be used to reconstruct the main features of the Galactic matter distribution. The similarities between the model and the actual Galaxy are sufficient to conclude on the possibilities to reconstruct the matter distribution.

However, an interesting follow-up on this work would be to apply the developed method on the already existing, more realistic, Gaia mock catalogue created by Robin et al. (2012). This simulated stellar catalogue differs from the in this work created datasets in that for example both a thin and a thick disk are included and that a more accurate distribution of different spectral types have been applied. Before applying the method to actual, real Milky Way data, it will be important to have established all the properties of the method for it to be possible to assess which part of the estimated mass density is actual matter distribution and not an artefact from e.g., surface fitting.

The strength of the model used in this work lies in its simplicity and easy applicability to astrometric data. Compared to the other models mentioned in the introduction (Chapter 1), this way of estimating the Galactic mass distribution is simpler and less computationally demanding, but might also be less accurate.

Detecting dark matter Whether the dark matter density in the Solar neighbourhood has been detected or not is a disputed issue. Moni Bidin et al. (2012) claim they cannot detect any local dark matter as they find \( \rho_{0,DM} = 0 \pm 0.001 \, M_\odot \, \text{pc}^{-3} \) from data of \( \sim 400 \) red giant stars in the thick disk. Using the same data, Bovy & Tremaine (2012) in turn claim to disprove the results of Moni Bidin et al. (2012), when by using another technique they find a local dark matter density of \( \rho_{0,DM} = 0.008 \pm 0.003 \, M_\odot \, \text{pc}^{-3} \). Garbari et al. (2011) also investigated the dark matter density in the Solar neighbourhood and they did so by applying a similar method as described in this work, first to the data from Hipparcos and then to the data from the Sloan Digital Sky Survey (SDSS). They find from the Hipparcos data the local dark matter density of \( \rho_{0,DM} = 0.033^{+0.008}_{-0.009} \, M_\odot \, \text{pc}^{-3} \). Garbari et al. (2011) blame the difference on the inaccurate measurements of the tracer stars. Thus, current estimates of the local dark matter distribution range from 0 to 30% of the total local mass density of \( \rho_0 = 0.102 \pm 0.010 \, M_\odot \, \text{pc}^{-3} \) estimated from Hipparcos data (Holmberg & Flynn, 2000).
Considering the high precision with which the method can estimate the mass density in the Solar neighbourhood (~ 1%), it is probable that the Solar neighbourhood dark matter distribution will be possible to determine. However, when it comes to these high precisions, what actually might limit the dark matter estimation could be the estimation of the present visible matter which is needed to subtract from the total estimation of mass density to achieve the dark matter density. This estimation of the visible matter is coarse as the abundance of invisible baryonic mass such as brown dwarfs and free-floating planets is not well known.

In Fig. 6.1, the contribution of the halo component compared to the total mass density in the potential model of Paczyński (1990) is shown. In the model, the halo has a mass density in the Solar neighbourhood of $\rho_{\odot, DM} \approx 0.0066 \, M_{\odot} \, pc^{-3}$ and thus contributes with about 4% of the total mass density in the region. Given that this value is very similar to the estimations of the local dark matter density described above, imagine that the whole halo in the Paczyński (1990) model consists of dark matter and that the configuration of the potential model is how the Milky Way looks like. As mentioned above, the region with most accurately probed mass density is within $6 \lesssim R \lesssim 9$ kpc, $|z| \lesssim 50$ pc and $x \gtrsim 5.5$ kpc with the precision of a few percent. From Fig. 6.1 it can be found that the dark matter concentration in this region constitutes between about 2.5% and 4.6% of the total mass density. With the accuracy of also a few percent the possibility to probe the dark matter distribution with this method is small for the current status of the method, but by a minor improvement, such as e.g., a more suitable grid or slightly better adapted surface fit parameters, the dark matter distribution in the region will become accessible. Already with the current method, the dark matter distribution could be possible to probe if the densities and contributions are tweaked with the more recent estimates of $\rho_{\odot, DM} = 0.008 \pm 0.003 \, M_{\odot} \, pc^{-3}$ (Jungman et al., 1996; Bovy & Tremaine, 2012) together with the total local mass density from Hipparcos given above. Then the contribution of dark matter in the Solar neighbourhood would be $7.3^{+4.1}_{-3.4}$%.

Figure 6.1 shows also how the contribution of the halo rapidly increases with height above the Galactic disk. This offers the opportunity to detect dark matter with the method, even though the errors of the method are large in this region.

Also, what should be kept in mind is that in the real Milky Way there might be a window of low extinction (which are not included in the smooth extinction model in this work) where the measurement errors would decrease drastically and the precision would reach lower values so that the dark matter could be probed. To be able to probe the dark matter structure in low extinction windows requires that the sub-structure of dark matter fits within the window. Diemand et al. (2008) suggest that the smallest dark matter structures present today would still be the primordial dark matter structures of Earth-mass and Solar system size. If this is the case, the dark matter sub-structure fits well into the available extinction windows, e.g., Baade’s window (Stanek, 1996).

Worth to mention is the theory about the Milky Way dark disk. This disk of higher concentration of dark matter than in the surrounding halo is believed to have formed from the accretion of dark matter during minor mergers.

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1The standard halo model, SHM (Jungman et al., 1996), is a model where the popular NFW dark matter halo profile is applied.
mergers between the Milky Way and satellite galaxies earlier in the history of the Galaxy (Read et al., 2008; Lake, 1989). Read et al. (2008) use simulations to estimate how often satellite galaxies merge with Milky Way-like galaxies and examine the accreted material from such a process. They find that the potential dark disk of the Milky Way should have a dark matter density of $\rho_{\text{dark disk}} = (0.25 \text{ to } 1) \rho_{\text{dark halo}}$, where $\rho_{\text{dark halo}}$ refers to the mass density of the dark halo the Milky Way is assumed to be equipped with.

Assume now that the Milky Way does have a dark disk of the type described above and that the dark matter halo is like the standard halo model (SHM, Jungman et al., 1996) with the local dark matter density of $\rho_{\odot, \text{DM}} = 0.008 \text{ M}_\odot \text{ pc}^{-3}$. Then, the contribution of dark matter (dark disk plus halo) in the Solar neighbourhood would be between roughly 10 and 15%. If dark matter constitutes this much of the local mass density, it will be possible to probe its distribution using the technique presented in this work.

**Improving the data** Not to be forgotten is that in this work only the limiting factors of Gaia is considered, while in reality the stars in the Milky Way can be observed with other telescopes as well, data which can be used to increase the accuracy of the Gaia measurements. 4MOST is a spectroscopic survey under development at the ESO telescope VISTA which aims to yield about 20 million spectra of resolution $R \sim 5000$ and about 2 million spectra with resolution $R \sim 20000$, which can be used to complement the radial velocities of the corresponding Gaia observations (de Jong et al., 2012). The Gaia-ESO survey will operate at the VLT FLAMES instrument and it is created to complement the Gaia satellite with high resolution spectra for about 100 000 stars which also will be used to improve the radial velocities (Gilmore et al., 2012). Combining the Gaia data with the Spitzer data of RR Lyrae stars has been estimated to improve the distance measurements from limited to about 10 kpc out to 100 kpc (Price-Whelan & Johnston, 2013).

The combination of the Gaia data with additional data will in the above described way allow for even more accurate measurements in the Gaia catalogue. Therefore, the measurement errors and extinction damage applied to the simulated data in this work might end up exaggerated. The errors in the mass density estimations in this work could therefore be interpreted as upper limits.
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47
Appendix A

Full derivation of expression for mass density

This appendix gives the full derivation of the expression for mass density belonging to Chapter 4.

A.1 Derivation of the continuity equation

The continuity equation, Eq. [3.1], can be found by considering a flow of particle through phase space, that is particles both moving and which have an acceleration throughout time. A small imaginary box in phase space has the volume $dV = dR d\theta dz du dv dw$. Imagine the flow in one direction, say spatial, so that particles move in through the surface $A$ and out through the surface $B$. The net flow of particles in the direction is then given by the change of the velocity in that direction. Let the considered coordinate be the cylindrical radius $R$. Then the net flow of particles is written

$$\left( f_B \dot{R}_B - f_A \dot{R}_A \right) d\theta dz du dv dw = \frac{\partial (f \dot{R})}{\partial R} dR d\theta dz du dv dw,$$

(A.1)

where the definition of derivative is used. Combining similar equations for all six phase space coordinates leads to the change of the distribution function with time

$$\frac{\partial (f \dot{R})}{\partial R} + \frac{\partial (f \dot{\theta})}{\partial \theta} + \frac{\partial (f \dot{z})}{\partial z} + \frac{\partial (f \dot{u})}{\partial u} + \frac{\partial (f \dot{v})}{\partial v} + \frac{\partial (f \dot{w})}{\partial w} = -\frac{\partial f}{\partial t}.$$  

(A.2)

Incorporating the definition of the cylindrical velocities

$$\begin{cases} u = \dot{R} \\ v = R \dot{\theta} \\ w = \dot{z} \end{cases}$$

(A.3)

leads to the continuity equation.

A.2 Acceleration in cylindrical coordinates

The definition of potential is given in Eq. [3.2]. In Cartesian coordinates the equation spelled out corresponds to the following

$$\begin{cases} \ddot{x} = -\frac{\partial \Phi}{\partial x} \\ \ddot{y} = -\frac{\partial \Phi}{\partial y} \\ \ddot{z} = -\frac{\partial \Phi}{\partial z}. \end{cases}$$

(A.4)
Using the conversion between the cylindrical and Cartesian coordinates the derivatives of the potential with respect to the cylindrical coordinated can be found to be

$$\begin{align*}
\frac{\partial \Phi}{\partial R} &= \frac{\partial \Phi}{\partial x} \frac{\partial x}{\partial R} + \frac{\partial \Phi}{\partial y} \frac{\partial y}{\partial R} = -\ddot{x} + \ddot{y} \\
\frac{\partial \Phi}{\partial \theta} &= \frac{\partial \Phi}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial \Phi}{\partial y} \frac{\partial y}{\partial \theta} = \ddot{x} - \ddot{y}x \\
\frac{\partial \Phi}{\partial z} &= \frac{\partial \Phi}{\partial z}.
\end{align*}$$

(A.5)

The velocity components in cylindrical coordinates are with conversion between cylindrical and Cartesian coordinates written as

$$\begin{align*}
u &= \frac{\partial R}{\partial t} = \dot{x} + \dot{y} \\
v &= R \frac{\partial \theta}{\partial t} = \dot{x} - \dot{y} \\
w &= \frac{\partial z}{\partial t},
\end{align*}$$

(A.6)

while the acceleration components can be written with Cartesian coordinates and together with Eq. (A.6) to be

$$\begin{align*}
\frac{\partial u}{\partial t} &= \frac{v^2}{R} + \frac{x\ddot{x} + y\ddot{y}}{R} \\
\frac{\partial v}{\partial t} &= \frac{\dot{x}x - \dot{y}x}{R} - \frac{uv}{R} \\
\frac{\partial w}{\partial t} &= \ddot{z}.
\end{align*}$$

(A.7)

Now combining Eqs. (A.5) and (A.7) the acceleration in cylindrical coordinates (when the acceleration is caused by a potential) is described as in Eq. (3.3).

### A.3 From collisionless Boltzmann equation to the second Jeans equations

The collisionless Boltzmann equation, Eq. (3.4), describes the motion of particles distributed in the way the distribution function, $f$, is constructed as they move in the potential, $\Phi$, with time. To be able to evaluate the collisionless Boltzmann equation, tricks are needed and that is what the Jeans equations are all about. The first Jeans equation is the equation achieved when the collisionless Boltzmann equation is integrated over all velocity components; $du \, dv \, dw$. Before it is presented it is important to see that the number density $n$ is defined as the number of particles per spatial volume, which can also be written as the integral of the distribution function over the velocity components as in Eq. (3.8). Further, the following relations will also be useful

$$\begin{align*}
n(u) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u f \, du \, dv \, dw \\
n(v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v f \, du \, dv \, dw \\
n(w) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w f \, du \, dv \, dw,
\end{align*}$$

(A.8)

where the angle brackets $\langle \rangle$ represent the mean of the value inside them. Then, by integrating Eq. (3.4) over $du \, dv \, dw$, the first Jeans equation is found

$$\frac{\partial n}{\partial t} + \frac{\partial (n(u))}{\partial R} + \frac{1}{R} \frac{\partial (n(v))}{\partial \theta} + \frac{\partial (n(w))}{\partial z} + \frac{n(u)}{R} = 0,$$

(A.9)
where it has been used that the derivatives with respect to the time or position can be moved outside the integrals so that the first term becomes
\[
\iiint \frac{\partial f}{\partial t} \, du \, dv \, dw = \frac{\partial}{\partial t} \iint f \, du \, dv \, dw = \frac{\partial n}{\partial t},
\]
(A.10)
and the second, third and forth terms are found through the following reasoning
\[
\iiint u \frac{\partial f}{\partial R} \, du \, dv \, dw = \frac{\partial}{\partial R} \iint u \, f \, du \, dv \, dw = \frac{\partial (n(u))}{\partial R},
\]
(A.11)
where the chain rule has been used
\[
\frac{\partial}{\partial R} (u f) = f \frac{\partial u}{\partial R} + u \frac{\partial f}{\partial R} = u \frac{\partial f}{\partial R}.
\]
(A.12)
Concerning the terms including derivatives of the potential, these derivatives can be moved outside of the integral and then the fact that the distribution function is zero at infinity is used to set the terms to zero
\[
\iiint \frac{\partial \Psi}{\partial R} \frac{\partial f}{\partial u} \, du \, dv \, dw = \frac{\partial \Psi}{\partial R} \iint \left( \int \frac{\partial f}{\partial u} \, du \right) \, dv \, dw = \frac{\partial \Psi}{\partial R} \iint \int_{u=-\infty}^{u=\infty} f \, dv \, dw = 0.
\]
(A.13)
In this derivation of the Jeans equations, due to the cylindrical coordinates, terms including velocities and derivatives of the distribution function with respect to velocities appear. In the integration of these terms, in addition to the reasoning in Eq. (A.13) the following partial integration is used
\[
\iiint \left( \int u \frac{\partial f}{\partial u} \, du \right) \, dv \, dw = \iiint \left[ u f \int_{u=-\infty}^{u=\infty} - \int f \, du \right] \, dv \, dw = -\iiint f \, du \, dv \, dw = -n,
\]
(A.14)
where in the second equality the first term can be set to zero as \(f\) goes to zero at \(\pm \infty\).

The second Jeans equations are three equations found from multiplying the collisionless Boltzmann equation, Eq. (3.4), with each velocity components and integrating over all velocity components. To find these equations yet another relation similar to Eq. (A.8) should be introduced
\[
n(uv) = \iiint u v f \, du \, dv \, dw.
\]
(A.15)
Otherwise, the relations presented in Eqs. (3.8), (A.8), (A.10), (A.11), (A.13) and (A.14) are enough to derive the second Jeans equations given in equations Eqs. (3.5), (3.6) and (3.7).

A.4 Deriving the mass density expression

To go from the second Jeans equations, Eqs. (3.5), (3.6) and (3.7), to the mass density expression in Eq. (3.14) requires differentiation with respect to the spatial coordinates and combining with the Laplacian of the potential, Eq. (3.10). This because Poisson’s equation, Eq. (3.9), is used.

The first term in the Laplacian in Eq. (3.10) can be found directly from Eq. (3.5) to be
\[
\frac{1}{R} \frac{\partial^2 \Phi}{\partial R^2} = -\frac{1}{R} \left( \frac{\langle u \rangle}{\partial t} \frac{\partial \ln n}{\partial t} + \frac{\langle u^2 \rangle}{\partial R} \frac{\partial \ln n}{\partial R} + \frac{\langle u \rangle \partial \ln n}{\partial R} + \frac{\langle u^2 \rangle \partial \ln n}{\partial R} + \frac{\langle u \rangle}{R} \frac{\partial \langle u \rangle}{\partial R} + \frac{\langle u \rangle}{R} \frac{\partial \langle u^2 \rangle}{\partial R} \right)
\]
(A.16)
Differentiating Eq. (3.5) with respect to \(R\) gives the second term in Eq. (3.10) to be
\[
\frac{\partial^2 \Phi}{\partial R^2} = -\left( \frac{\langle u \rangle}{\partial R} \frac{\partial \ln n}{\partial R} + \frac{\langle u^2 \rangle}{\partial R} \frac{\partial \ln n}{\partial R} + \frac{\langle u \rangle \partial \ln n}{\partial R} + \frac{\langle u^2 \rangle \partial \ln n}{\partial R} + \frac{\langle u \rangle}{R} \frac{\partial \langle u \rangle}{\partial t} + \frac{\langle u \rangle}{R} \frac{\partial \langle u^2 \rangle}{\partial t} \right)
\]
(A.17)
The third term in Eq. (3.10) is found by differentiating Eq. (3.6) with respect to θ

\[
\frac{1}{R^2} \frac{\partial^2 \Phi}{\partial \theta^2} = -\frac{1}{R} \left( \frac{\partial \langle v \rangle}{\partial \theta} \frac{\partial \ln n}{\partial t} + \langle v \rangle \frac{\partial^2 \ln n}{\partial \theta \partial t} + \frac{\partial^2 \langle v \rangle}{\partial \theta^2} \frac{\partial \ln n}{\partial \theta} + \langle uv \rangle \frac{\partial^2 \ln n}{\partial \theta \partial R} \right) \\
+ \frac{1}{R} \frac{\partial \langle v^2 \rangle}{\partial \theta} \frac{\partial \ln n}{\partial t} + \langle v^2 \rangle \frac{\partial^2 \ln n}{\partial \theta^2} + \frac{1}{R} \frac{\partial \langle v \rangle}{\partial \theta} \frac{\partial \ln n}{\partial \theta} + \langle \langle v \rangle \frac{\partial^2 \ln n}{\partial \theta \partial \theta} \right) \\
+ \frac{\partial^2 \langle v \rangle}{\partial \theta \partial \theta} + 2 \frac{\partial \langle uv \rangle}{\partial \theta}. \tag{A.18}
\]

Finally, the fourth term in Eq. (3.10) is found by differentiating Eq. (3.7) with respect to z

\[
\frac{\partial^2 \Phi}{\partial z^2} = -\left( \frac{\partial \langle w \rangle}{\partial z} \frac{\partial \ln n}{\partial t} + \langle w \rangle \frac{\partial^2 \ln n}{\partial z \partial t} + \frac{\partial^2 \langle w \rangle}{\partial z^2} \frac{\partial \ln n}{\partial z} + \langle uw \rangle \frac{\partial^2 \ln n}{\partial z \partial R} \right) \\
+ \frac{1}{R} \frac{\partial \langle w \rangle}{\partial z} \frac{\partial \ln n}{\partial \theta} + \langle \langle w \rangle \frac{\partial^2 \ln n}{\partial z \partial \theta} + \frac{\partial \langle w^2 \rangle}{\partial z} \frac{\partial \ln n}{\partial z} + \langle \langle w^2 \rangle \frac{\partial^2 \ln n}{\partial z^2} \right) + \frac{\partial \langle uw \rangle}{\partial z}. \tag{A.19}
\]

Now, combining the Eqs. (A.16), (A.17), (A.18) and (A.19) into Poisson’s equation, Eq. (3.9), through Eq. (3.10).
Appendix B

Finding values for $\sigma_{v_r}$

In Chapter 2, the uncertainties for the radial velocity measurement for different types of stars is considered. The dispersion in this measurement is given by Eq. (2.6) and this equation includes two constants, $a$ and $b$ which are different for different types of stars due to their different properties. At Gaia Science Performance web page, these constants are given for a few types of stars. To proceed investigating which the best suited tracer star would be in this work, these constants are needed for more spectral types than the given at Gaia Science Performance. Therefore, here, it is explained how the values for these constants are estimated for the other included spectral types in the investigation.

At Gaia Science Performance, the value for $a$ is given between 0.90 and 1.15 for all the different spectral types. Because these values are reasonably constant, the other stars used in this work are given $a$-values of 1.15. The $b$-values on the other hand, have more varying numbers. In Fig. B.1 the 10-logarithm of the given $b$-values are plotted with the colour $(V − I_C)$ as black dots. As can be seen, the trend is roughly linear and therefore the $b$-values for the other spectral types are found from the linear fit (they are marked with red dots). In the figure, the dots with a plus on top correspond to the spectral types included in this work and the corresponding values are given in Table B.1.

<table>
<thead>
<tr>
<th>Spectral type</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0V*</td>
<td>0.90</td>
<td>50.0</td>
</tr>
<tr>
<td>B5V*</td>
<td>0.90</td>
<td>26.0</td>
</tr>
<tr>
<td>A0V*</td>
<td>1.00</td>
<td>5.50</td>
</tr>
<tr>
<td>A5V*</td>
<td>1.15</td>
<td>4.00</td>
</tr>
<tr>
<td>F0V*</td>
<td>1.15</td>
<td>1.50</td>
</tr>
<tr>
<td>F5V</td>
<td>1.15</td>
<td>1.55</td>
</tr>
<tr>
<td>G0V*</td>
<td>1.15</td>
<td>0.70</td>
</tr>
<tr>
<td>G5V*</td>
<td>1.15</td>
<td>0.60</td>
</tr>
<tr>
<td>K0V*</td>
<td>1.15</td>
<td>0.50</td>
</tr>
<tr>
<td>K5V</td>
<td>1.15</td>
<td>0.13</td>
</tr>
<tr>
<td>M0V</td>
<td>1.15</td>
<td>0.020</td>
</tr>
<tr>
<td>M5V</td>
<td>1.15</td>
<td>0.00059</td>
</tr>
<tr>
<td>G5III</td>
<td>1.15</td>
<td>0.39</td>
</tr>
<tr>
<td>K0III</td>
<td>1.15</td>
<td>0.27</td>
</tr>
<tr>
<td>K5III</td>
<td>1.15</td>
<td>0.026</td>
</tr>
<tr>
<td>M0III</td>
<td>1.15</td>
<td>0.023</td>
</tr>
<tr>
<td>M5III</td>
<td>1.15</td>
<td>0.00011</td>
</tr>
</tbody>
</table>

* The spectral types given already at Gaia Science Performance.

Figure B.1: How the $b$-values of more spectral types than given on the Gaia Science Performance webpage are found through linear fitting. Colour index $V - I_C$ for the different spectral types can be found in Table 4.1. (The relation is expected to be linear as the uncertainty in Eq. (2.6) is based on photon noise and the linearity can then be deduced from the relations between the bands found in Jordi et al. (2010).)
Appendix C

Star investigation with the Besançon model

In Chapter 4 Sect. 4.1.3.2 it is briefly described how the Besançon model (Robin et al., 2003) is used to determine which spectral type is most appropriate as tracer star. Here, this investigation is described in more detail.

The Besançon model is a simulation of stars in an axisymmetric model of the Milky Way. Providing a direction in Galactic coordinates, the simulation returns information about the stars present in that region. As input parameters to the Besançon model comes, amongst other, the spectral types of stars to consider, maximum distance within which the investigation should be carried out, size of the field of view, ranges or limits in apparent magnitudes and colour indices and extinction. From the specified request, the answer comes in the shape of either tables which are also specifiable or a list of each star and its properties. The first alternative is much less memory craving and was therefore the choice made in this work. It would be more accurate to choose the list of stars, but the Besançon model is here only used for a rough estimation of the number of stars and the distribution of stars available for Gaia.

As the Besançon model is axisymmetric, only three directions in Galactic longitude were considered and they are $\ell = 0^\circ$, $90^\circ$ and $180^\circ$, assuming that the distribution of stars in the direction $\ell = 270^\circ$ would equal that in the direction of $\ell = 90^\circ$ due to axisymmetry. In Galactic latitude seven directions were considered and they are $b = 0^\circ$, $2^\circ$, $5^\circ$, $10^\circ$, $20^\circ$, $45^\circ$ and $90^\circ$. The reason for the smaller interval in the small Galactic latitude is chosen because of the Galactic disk being thin. From axisymmetry, the lower hemisphere is neglected as it is assumed to equal the corresponding directions in the upper hemisphere.

The spectral types considered in the investigation are the ones given in Table 4.1 (full ranges). The magnitude limits given were $0 < V < 23$ mag, the colour index $-4.5 < V - I_C < 4.5$ mag and the maximum distance was 30 kpc. A smooth extinction of $A_V = 0.7$ mag/kpc was given. Depending on the abundance of the spectral type of star, the field of view was varied between $0.5^\circ$ and $100^\circ$.

Magnitudes were given in the Johnson $V$ magnitude instead of the Gaia $G$ magnitude in the input for the Besançon model. This because the option of Gaia $G$ was not available and a conversion to Gaia $G$ after achieving the results would not be accurate as the output from the Besançon model came as statistics in tables and thus the distance to individual stars was known with a precision of a kpc. Therefore, the magnitude limits were set to be $6 < V < 20$ mag for the Astrometric Field and $6 < V < 16$ mag for the Radial Velocity Spectrometer corresponding almost to the actual magnitude limits (see Sect. 2.1).

To find the total amount of stars visible of a specific type, the density of stars found per square degree was used in a bigger area in the same region. These areas are displayed in Fig. C.1. As can be seen, the regions are smaller towards the Galactic centre and the reason for this is that the density of stars is expected to fall off rapidly when moving away from the very centre. The number of stars found by investigating in this way is given in Table 4.1. It should be noted that the amount of stars available for the radial velocity spectrometer is estimated to be roughly half of the 150 million that Robin et al. (2012) found. This could be an effect of the magnitude limits not being the correct. However, the total number of stars estimated to be visible with the astrometric field corresponds to the 1.1 billion stars Robin et al. (2012) predict.

Which region of the Galaxy a particular spectral type of star typically inhabits can very roughly be estimated using the data obtained from the Besançon model. The distribution found shows that all stars are concentrated to the Galactic disk.
(a) The four directions $\ell = 0^\circ, 90^\circ, 180^\circ$ and $270^\circ$ are used in the Besançon model. As the stellar density in the central region can be expected to be very high and with a quickly falling gradient, the approximated areas towards the centre of the Galaxy span an angle of $50^\circ$ in Galactic longitude, while the opposite direction has an angle of $130^\circ$ and the side directions have angles of $90^\circ$. The top circle of Galactic latitude is approximated with the value at the pole.

(b) Side-view of the viewing directions, here showing also the Galactic latitude directions $b = 0^\circ, 2^\circ, 5^\circ, 10^\circ, 20^\circ, 45^\circ$ and $90^\circ$ and also symmetrically on the lower hemisphere. The areas which use these values to approximate a stellar density are restricted with $b = 1^\circ, 3^\circ, 7^\circ, 13^\circ, 30^\circ$ and $60^\circ$.

Figure C.1: A celestial sphere showing directions on the sky investigated with the Besançon model with red dots while the bigger black dot corresponds to the position of the Solar system. The direction of the Galactic centre is in the direction of the positive $x$-axis. The regions restricted with black lines signify the areas on the sky approximated to have the same stellar density as measured in the red dot when estimating the total amount of observable stars.
Appendix D

Interpretation of the extinction model of Amôres & Lépine (2005)

In this appendix, the smooth extinction model based on hydrogen observations in the Milky Way and created by Amôres & Lépine (2005) is interpreted by the author of this work.

The number density, \( n_H \), of the neutral and molecular hydrogen is assumed to have the following shape

\[
n_H(R, z) = c \cdot \exp \left\{ -\frac{R}{a} - \left( \frac{b}{R} \right)^2 \right\} \cdot \exp \left\{ - \left( \frac{z}{1.2 \cdot k \cdot \exp \{10^{-4} R\}} \right)^2 \right\}, \tag{D.1}
\]

where \( a, b, c \) and \( k \) are constants which vary between neutral and molecular hydrogen as shown in Table D.1. \( R \) and \( z \) are given in pc and the number density appears then in the unit the constant \( c \) is given in. It should be added that in this work, the Sun is assumed to lie in the Galactic plane \( (z = 0) \) and therefore one parameter is removed from the original expression of Amôres & Lépine (2005).

Table D.1: Constants for neutral and molecular hydrogen correlated to the extinction model by Amôres & Lépine (2005).

<table>
<thead>
<tr>
<th>Constant</th>
<th>HI</th>
<th>H₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>7 kpc</td>
<td>1.2 kpc</td>
</tr>
<tr>
<td>( b )</td>
<td>1.9 kpc</td>
<td>3.5 kpc</td>
</tr>
<tr>
<td>( c )</td>
<td>0.7 cm(^{-3})</td>
<td>58.0 cm(^{-3})</td>
</tr>
<tr>
<td>( k )</td>
<td>81 pc</td>
<td>45 pc</td>
</tr>
</tbody>
</table>

As the column densities of the hydrogen are used to find the extinction in every direction, they are calculated by integrating the number density along the line of sight\(^1\)

\[
N_H(x, y, z) = \int_0^r n_H(R, z) \, dr',
\tag{D.2}
\]

where \( r \) is the end of the column (i.e., the distance to the star) seen from the Solar system in pc and \( r' \) the radius in Heliocentric, spherical coordinates in pc. The extinction is correlated to the column densities in the following way

\[
A_V(x, y, z) = \gamma(R) \{ N_{HI}(x, y, z) + 2N_{H₂}(x, y, z) \},
\tag{D.3}
\]

where \( \gamma(R) \) is a function giving the Galactocentric, radial decrease to expect from the fact that the Galaxy gets more and more dilute towards the edges. It is connected to the Galactocentric radius in the following way

\[
\gamma(R) = \frac{k_1}{\sqrt{R/1000}},
\tag{D.4}
\]

\(^1\)There are three different coordinate systems present in this equation; \( x, y, z \) signifies the Galactocentric, Cartesian coordinate system, \( r \) and \( r' \) refer to the Heliocentric spherical coordinates and \( R \) and \( z \) correspond to the Galactocentric, cylindrical coordinates.
where $R$ is given in pc and $k_1$ is a constant. For the Sun’s distance, $R_\odot = 8$ kpc, $\gamma$ has been determined by Bohlin et al. (1978) to be $\gamma_0 = 5.3 \cdot 10^{-22}$ mag cm$^2$ and by applying the value to Eq. (D.4), the constant $k_1$ can be found ($k_1$ does not correspond to $k$).

In Eq. (D.3), values already integrated over the line of sight are multiplied by $\gamma(R)$, which is dependent of $R$. This fact leads to an unphysical decrease in extinction along some lines of sight. The regions involved in this work are however not affected. This can be seen by comparing the extinction model seen in Fig. 4.5 with the to Gaia visible regions in Fig. 4.6.
Appendix E

Including spiral arms

To test the limits of the method described, it has also been used on two additional datasets with non-axisymmetric structures and which thus are more realistic. However, in non-axisymmetric structures, the system can no longer be assumed to be static due to rotation. This would require that the time-dependent terms in Eq. (3.11) were estimated as well. This is something which will not be possible in the case of Gaia, as the mission lifetime is effectively instantaneous compared to the dynamical time with which the Galaxy evolves. Because of the lack of the time derivatives, the method is not expected to give the correct outcome when applied to the data created in non-axisymmetric structures.

The first consists of stars simulated in the potential model of Paczyński (1990) but where the central bulge has been replaced by a bar described by the triaxial model of Long & Murali (1992). As expected, the bar did not show any signs of density differences in the region of the grid used in this work (see Fig. 4.7) as it is occupying a smaller central region. Therefore, this model and the corresponding results are not included in this chapter.

Instead, this appendix focuses on the data produced in the second potential model, which consists of the axisymmetric model of Paczyński (1990) on top of which a spiral density pattern has been superposed. In this case, the density waves are visible and therefore here, the model will be described and the results presented. As the validity of the results is debatable due to the predicted non-applicability of the Jeans equations on this non-static case, this is presented as a first attempt of applying the method to this type of structure and is not included in the main part of the thesis.

E.1 Potential model with spiral arms

A simple, analytic model for spiral arms is used to resemble the actual structure in the Milky Way. The Galaxy is known to have at least four arms (Taylor & Cordes 1993; Georgelin & Georgelin 1976; Drimmel 2000), but as two of them are very prominent (Drimmel 2000), the structure used here will be that of a grand design galaxy with only two arms present. They have a density amplitude of \( \rho_0 = 0.025 \, \text{M}_\odot \, \text{pc}^{-3} \) at the reference radius \( R_0 = 8 \, \text{kpc} \) (Feng & Bailer-Jones 2013).

The model used is developed initially by Cox & Gómez (2002), but the interpretation of the model by Feng & Bailer-Jones (2013) is simpler and still adapted to the spiral pattern of the Milky Way, so therefore is their version of the model applied here. The potential model looks like the following

\[
\Phi_s = -\frac{4 \pi G H K_1 D_1}{\rho_0} \exp \left( - \frac{R - R_0}{R_s} \right) \times \cos(N \left[ \theta - \theta_s(R,t) \right]) \left[ \text{sech} \left( \frac{K_1 z}{\beta_1} \right) \right]^{\beta_1},
\]

where \( R, \theta, \) and \( z \) correspond to the Galactocentric, cylindrical coordinate system used, \( G \) to the gravitational constant, \( H \) to the scale height of the spiral density, \( \rho_0 \) to the earlier mentioned midplane arm density at the reference radius \( R_0 \), \( R_s \) is the drop-off radial length of the density amplitude or the arms and \( N \) the number of arms. \( \theta_s(R,t) \) describes the shape of the arms and how they rotate (they are set rotating rigidly and trailing) in the following way

\[
\theta_s(R,t) = \theta_s(R) + \Omega_p t
\]

\[
\theta_s(R) = -\left( \alpha \log(R/R_{\text{min}}) + \theta_{\text{min}} \right),
\]

\( \alpha \), \( \Omega_p \), and \( \theta_{\text{min}} \) are the arm amplitude, the characteristic rotation frequency, and the minimum arm amplitude at the reference point, respectively.
Table E.1: Values for the constants in the potential for spiral arms given in Eq. (E.1).

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern speed</td>
<td>$\Omega_p$</td>
<td>20 km/s/kpc</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>$\alpha$</td>
<td>4.25 rad</td>
</tr>
<tr>
<td>Inner radius</td>
<td>$R_{\text{min}}$</td>
<td>3.48 kpc</td>
</tr>
<tr>
<td>Azimuth at $R_{\text{min}}$</td>
<td>$\theta_{\text{min}}$</td>
<td>0.26 and 3.40 rad</td>
</tr>
<tr>
<td>Arm extent</td>
<td></td>
<td>6.0 kpc</td>
</tr>
<tr>
<td>Number of arms</td>
<td>$N$</td>
<td>2</td>
</tr>
<tr>
<td>Mass density at $R_0$</td>
<td>$\rho_0$</td>
<td>0.025 $M_\odot$ pc$^{-3}$</td>
</tr>
<tr>
<td>Reference (fiducial) radius</td>
<td>$R_0$</td>
<td>8 kpc</td>
</tr>
<tr>
<td>Density drop-off radius</td>
<td>$R_s$</td>
<td>7 kpc</td>
</tr>
<tr>
<td>Scale height of spiral pattern</td>
<td>$H$</td>
<td>0.18 kpc</td>
</tr>
</tbody>
</table>

with $\Omega_p$ the pattern speed of the arms, $t$ the time coordinate, $\alpha$ the pitch angle of arms, $R_{\text{min}}$ the inner radius at which the arms should begin and $\theta_{\text{min}}$ the azimuth at the inner radius of the particular arms. In Eq. (E.3) an additional minus sign has been added compared to the original configuration for the model to fit the current coordinate system$^1$. $K_1$, $\beta_1$ and $D_1$ are constants defined as the following

\[
K_1 = \frac{N}{R \sin(\alpha)} \quad (E.4)
\]

\[
\beta_1 = K_1 H (1 + 0.4 K_1 H) \quad (E.5)
\]

\[
D_1 = \frac{1 + K_1 H + 0.3 (K_1 H)^2}{1 + 0.3 K_1 H} \quad (E.6)
\]

The values used in Feng & Bailer-Jones (2013) are also applied here and are therefore repeated in Table E.1 for references to the different values the reader is encouraged to examine Feng & Bailer-Jones (2013).

To apply the spiral pattern to the Galaxy model, Eq. (4.1) is simply modified into

\[
\Phi = \Phi_b + \Phi_d + \Phi_h + \Phi_s, \quad (E.7)
\]

and as Feng & Bailer-Jones (2013) use an axisymmetric potential model similar to the potential by Paczyński (1990) described in Sect. 4.1.1, it is used also here. The axisymmetric model of Feng & Bailer-Jones (2013) is not applied, because if the axisymmetric base would be changed, the comparison between the results from the axisymmetric potential model and the model with spiral arms would not be as useful. Note that even though there should be two arms present in the galaxy, only one potential of the shape in Eq. (E.1) is added, here using the azimuth angle $\theta_{\text{min}} = 0.26$ rad (see Table E.1). Note also that the spiral pattern potential is cut off after it has reached its full extent, which in this case happens at $R \approx 9.5$ kpc.

The evolution of the phase space coordinates with time is seen in Fig. E.1 where phase mixing is visible also in this dataset as in the case of the axisymmetric potential (see Fig. 4.3). The spiral arms are clearly visible in the evolution plot for the azimuth angle, and what is also visible in this plot is the chaos the spiral arms cause to the stars as they mix efficiently with time. This means that the stars in the spiral potential are not located mainly in the spiral arms. This seem also to apply to the Milky Way where the young stars inhabit the spiral arms, but the older have a more uniform distribution over the azimuth angle. The efficient mixing due to the spiral arms is also visible in the radius $R$ and the radial velocity $u$ and tangential velocity $v$, where compared to Fig. 4.3 the distributions mix faster and are also broader (more extreme values are achieved) in the case of the velocities.

In the simulation run in the potential described above, the integration time is set to about 550 Myr, which might seem from analysing Fig. E.1 like a too short time due to the still on-going phase mixing, but the decision was limited by the much longer computation time the new potential came with. The threshold time was set to about 300 Myr. The simulation has in total 13 801 932 stars, a number of which far fewer stars can be used than in the case of the axisymmetric simulation as the stars now have been perturbed to move into the central regions of the galaxy. However, as the number can be doubled by mirroring the data over the disk due to the symmetry still present over the Galactic disk, the number increases again as is seen in the results in the following Sect. E.2.

$^1$This is for the arms to be trailing when the arms are moving in the direction defined as positive in the coordinate system.
Appendix E: Including spiral arms

Figure E.1: Evolution of the stellar distribution in the spiral arms potential during an integration of about 2 billion years. The spatial coordinates $R$, $\theta$ and $z$ are shown in the left column from top to bottom and the velocity coordinates in the right column with $u$, $v$ and $w$ from top to bottom. The dynamical mechanism phase mixing is visible in all the plots, here also in the azimuth angle $\theta$ where the over-density of two spiral arms and their rotation is visible. Phase mixing is visible through the change from the initial distribution to the final stabilised distribution.
E.2 Simulation results for spiral arms

When creating the simulation in the potential model equipped with spiral arms, stars were initialised with the same starting conditions as for the main simulation as described in Sect. 4.1.2. They were integrated for about 550 Myr as the potential is computationally heavy 2000 Myr was not possible time-wise. However, using the potential from Paczyński (1990) and integrating for 550 Myr has given rise to almost equivalent results as when the integration went on for 2000 Myr, therefore this investigation is carried out anyhow. After randomly picking each star’s final values, the azimuth coordinate \( \theta \) of the star was adjusted so that the spiral pattern potential would be oriented in the starting position. The spiral pattern itself was set rotating rigidly with the pattern speed given above. This is most probably not the way spiral arms behave, as the common belief is that the spiral arms are density waves (Lin & Shu, 1964).

Here, the results from first the full and perfect dataset is used in Sect. E.2.1. Then the most realistically perturbed data with measurement errors and extinction and using the cut off grid together with photometry as in the case of the dataset \( x_{\text{cut5phom,FBJ13}} \), but with this other initial simulation. This is presented in Sect. E.2.2. The properties of the datasets are also shown in Table E.2.

### E.2.1 perfect\_FBJ13

In Fig. E.3 it is seen, by comparing the left column of plots corresponding to the estimated values with the right column of plots showing the true mass density values, that even though it is no longer possible to reconstruct as it really is. However, a spiral pattern is clearly visible, even though it seems to have been shifted slightly in the counter-clockwise direction. Also, the estimated density fluctuations are not as large as expected. Both these artefacts might be caused by the surface fits producing the results no longer have sufficient amount of data and fit sloppily to what data there are.

As in this case, it is important to be able to resolve the structure in the Galactic plane, the resolution in the meridional plane is sacrificed in this section and only ten bins in the vertical direction were used between \( 0 < z < 1 \) kpc. This is Grid 4 in Fig. E.2. As is seen by the coarse results, this barely allows for resolution in the azimuth angle. To decrease the amounts of bins further in the vertical direction might cause too much inaccuracy as the spiral density pattern only is supposed to occupy a limited region in space.

### E.2.2 \( x_{\text{cut5phom,FBJ13}} \)

Figure E.3 shows the mass density estimation after extinction and measurement errors have been added to the data in the way described in Sect. 4.1.3. Also, the grid found most suitable in the results of the simulation in the axisymmetric model (see Sect. 5.3) has been adapted into Grid 5 seen in Fig. E.2.

In these plots it is more difficult to see the spiral structure, but still there are some regions of several bins seemingly more or less dense than the surrounding regions. Also, what should be considered when interpreting these results is that a weaker part of the spiral pattern seem to be situated exactly in the region where the grid is active. If this is not the case in reality, it might be that the spiral pattern would be possible to better discern. Considering the good agreement between the results in Fig. 5.1 compared to Fig. 5.6 the hope is not lost for detecting spiral wave pattern using the described technique.

### Table E.2: Corresponding table as Table 5.1 but for the case of the additional spiral pattern introduced.

<table>
<thead>
<tr>
<th>Name</th>
<th>( r &lt; 10 \text{ kpc} )</th>
<th>M.E.</th>
<th>Ext.</th>
<th>( x \gtrsim 5.5 \text{ kpc} )</th>
<th>( d_{\text{phom}} )</th>
<th>Grid</th>
<th>Nbr of stars</th>
</tr>
</thead>
<tbody>
<tr>
<td>perfect_FBJ13</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>4</td>
<td>20 806 620</td>
</tr>
<tr>
<td>( x_{\text{cut5phom,FBJ13}} )</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>5</td>
<td>3 394 774</td>
</tr>
</tbody>
</table>

Notes. The grid used is found in Fig. E.2.
Appendix E: Including spiral arms

(a) Grid 4: Grid limited by $4.5 < R < 12$ kpc and $|z| < 1$ kpc. 20 bins in $R$, 20 bins in $\theta$ and 10 (20 when doubled) bins in $z$. Used by the dataset perfect_fbj13.

(b) Grid 5: Grid limited by $R < 12$ kpc, $r < 10$ kpc, $x \gtrsim 5.5$ kpc and $|z| < 1$ kpc. 20 bins in $R$, 8 bins in $\theta$ (before the cuts there was 20) and 10 (20 when doubled) bins in $z$. Used by the dataset xc5phom_fbj13.

Figure E.2: Grid used for the simulations in the potential model with spiral arms.
Figure E.3: Results from the dataset with perfect data generated in the potential model which includes spiral arms. From left to right, top to bottom: 1. estimated mass density, 2. true mass density, 3. difference between estimated mass density and the mean of the estimated mass density over the azimuth angle, $\theta$, and 4. difference between true mass density and the mean of the true mass density over the azimuth angle, $\theta$. 20 bins in azimuth angle $\theta$, here is the mean of four bins in vertical direction shown and they are located between $|z| \lesssim 118$ pc. Location of where the spiral arms should be is marked with thick black lines in all panels.
Appendix E: Including spiral arms

Figure E.4: Results from the dataset with data perturbed by extinction and spiral arms, but assuming photometry can be used for some distances (as described in Sect. 2.1.5). The grid is divided in bins in radial ($R$), azimuth ($\theta$) and vertical ($z$) direction with an additional cut at $x \approx 5.5$ kpc to avoid badly estimated number density. Simulation created in the potential with spiral arms. From left to right, top to bottom: 1. estimated mass density, 2. true mass density, 3. difference between estimated mass density and the mean of the estimated mass density over the azimuth angle, $\theta$, and 4. difference between true mass density and the mean of the true mass density over the azimuth angle, $\theta$. 20 bins in azimuth angle $\theta$, here is the mean of four bins in vertical direction shown and they are located between $|z| \lesssim 118$ pc. Location of where the spiral arms should be is marked with thick black lines in all panels.
E.3 Discussion and conclusion

Primarily in Fig. E.3 but also to some extent in Fig. E.4 it can be seen that the non-axisymmetric spiral structure added to the axisymmetric potential is resolvable using the described method. The estimation is not completely as expected as the pattern seem to have rotated somewhat counter-clockwise and is not as dense as the potential describes. These features might have the explanation that the number of stars available are simply not large enough for the estimation to be accurate in all three space directions. Another reason could be that the integration time was not long enough and the stars have then not yet gone through phase mixing enough to represent the potential in a good way. A third explanation is correlated with the inapplicability of the Jeans equations and thus the expression of mass density in Eq. (3.11) on non-static systems during such a short period of time as is considered here.

Even though the spiral arms seem somewhat possible to probe in the results in Sect. E.2 it should not be forgotten that the Milky Way is thought to have more than two spiral arms. If there would be located less dense spiral pattern in between the spiral arms in this investigation, it is possible that this perturbation is no longer detectable. Another difference between the data and the actual Milky Way is that in the data the spiral arms have been rotating rigidly, which is probably not the case of the Milky Way spiral arms. How much this affects the data is not established.

To thoroughly test the method on non-axisymmetric structures, a dataset which is integrated for longer, maybe one billion years or longer as then the distribution in $R$ and $\theta$ seems to have stabilised somewhat (see Fig. E.1), and which includes more stars so that the surface fits become more accurate would be interesting to create.
Appendix F

Wordlist

This word list contains the explanations of words as interpreted by the author.

" See arcsecond.

ϖ See parallax.

μ Can refer to $10^{-6}$ (micro), the absolute value of the proper motion $\mu$ or the distance modulus.

μas Micro-arcsecond, that is $10^{-6}$ arcseconds.

$A_X$ See extinction.

Absolute magnitude The magnitude an object has if seen at a distance of 10 pc.

Apparent magnitude The magnitude an object has when observed.

arcmin See arcminute.

Arcminute A sixtieth of a degree. Abbreviated as arcmin or ‘.

arcsec See arcsecond.

Arcsecond A sixtieth of an arcminute. May be abbreviated to arcsec, as or “.

Astrometry The science in accurately measuring the positions of objects on the sky.

Astronomical unit (AU) The distance between the Sun and the Earth. 149 597 870 700 meters.

Axisymmetric Symmetric around one axis. In this case the $z$ axis is the one axisymmetry occurs around.

(Axisymmetric, sometimes also referred to as axially symmetric, but this term is not used in this work.)

AU See Astronomical unit.

Band, photometric Region of wavelength which a filter lets through. Transmission of the filter shows the band.

Binary star Two stars bound to each other with gravitational force.

Cepheid A pulsating star that has a relationship between its pulsation period and its luminosity causing the brightness of the star to change periodically. From this change, the distance to the star can be accurately determined.

Colour excess ($E_{X-Y}$) The excess of colour a star seems to have due to interstellar reddening. $E_{X-Y}$ is then the difference between the colour index of the photometric bands $X$ and $Y$, that is $X - Y$, as it appears to be after the light has been reddened and as it actually is.
**Colour index** \((X - Y)\) The difference between the measured magnitudes of two different photometric bands. It is written as \(X - Y\) if it compares the band \(X\) with the band \(Y\). In this work, the most commonly used colour index is the difference between the Johnson visible \(V\) band and the Johnson Cousins infrared \(I_C\) band, that is \(V - I_C\).

**Column density** The density measured per area of a column directed along the line of sight. The length of the column can be unknown, thus the measure in surface density.

**Declination** \((\text{dec or } \delta)\) Coordinate used to measure position of objects on the sky. Measured in degrees between \(-90^\circ\) and \(90^\circ\) where \(0^\circ\) corresponds to the equatorial plane.

**Distance modulus** The decrease in magnitude due to distance. See Eq. \((2.8)\). Notation is \(\mu\).

**Distribution function** Probability function describing the probability to find, in this case stars, at a specific location with a specific velocity.

**Epicycle/epicyclic orbit** When a star moves in the force field of a galaxy, with a velocity slightly deviating from the circular case, the star will be seen to have two periods. First, the period with which the star orbits the centre, second, the period with which it fluctuates around the circular velocity. This second motion is referred to as the epicycle and the star is said to move on an epicyclic orbit.

**Extinction** The decrease of magnitude due to scattering of light on interstellar dust. Notation used is \(A_X\) for the photometric band \(X\). The extinction used in this work is \(A_V\).

**Field of view (FOV)** The region on the sky at which you look/have pointed your telescope.

**Galactocentric** Centred on the Galactic centre.

**Galaxy** The Milky Way galaxy.

**Galaxy** A galaxy. That is a self-gravitating system of stars, gas and dust. There are galaxies shaped elliptically, in disks and irregularly, called thereafter. The disk galaxies may contain spiral arms and a central bar. Galaxies can contain supermassive black holes in the centre.

**Gaussian distribution** Very common probability distribution, used for e.g., measurement errors. Has the form \(f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}\), where \(\mu\) is the mean of the parameter \(x\) and the distribution \(f(x)\) has the standard deviation \(\sigma\).

**Globular cluster** Cluster of stars, which held together by the gravitational force. Typically old and remnants from the collapse of the protogalaxy.

**Heliocentric** Centred on the Sun.

**ICRS (International Celestial Reference System)** A coordinate system used to pinpoint the position of stars in.

**Interstellar reddening** The effect that interstellar dust is more keen to scatter bluer, short wavelengths than redder, long wavelengths.

**ISM (interstellar medium)** The medium between stars, typically consisting of gas and dust in galaxies.

**Line of sight (LOS)** The column of space located between you/your telescope and the astronomical target you want to look at. Might contain for example gas and dust which can distort the image of the target object.

**Magnitude** The brightness of an object. Measured in magnitudes. Bright objects have low magnitude, it can even be negative, while weak objects have high magnitude.

**mas** Milli-arcsecond, that is a thousandth of an arcsecond.

**Normal distribution** See Gaussian distribution.

**Oblate** Shaped as a discus.
**Appendix F: Wordlist**

**Parallax** The maximum angular difference an object is seen to have on the sky during the orbit of the Earth around the Sun. Used to estimate distance to an object in astrometry. Notation here is \( \varpi \).

**Parsec** The distance to a star which has a parallax of one arcsecond. The Solar neighbourhood is usually thought of as the sphere with radius 100 pc around the Sun. The distance to the Galactic centre is approximately eight kiloparsec (kpc) and the diameter of the Galactic disk is about 30 kpc.

**Pattern speed** The angular speed a pattern, such as a central bar or spiral arms rotate the galactic centre with.

**pc** See parsec.

**Phase space** A space defined by both position and velocity. In this work phase space usually refers to the six cylindrical coordinates \( R, \theta, z, u, v \) and \( w \), where the first three represent the position and the last three the velocity.

**Point spread function (PSF)** The image an imaging system manages to create of a point source. Different effects might affect the shape of the point spread function.

**Prolate** Cigar-shaped.

**Protogalaxy** A cloud of gas forming a galaxy.

**Right ascension (RA or \( \alpha \))** Coordinate to measure the position of an object on the sky. Measured from the vernal equinox and counter-clockwise around the Earth’s equator as seen when looking on the north pole. Measured in hours, minutes and seconds, one turn is then interpreted as 24 hours. The other coordinate is called declination.

**Rotation curve** Typically used in the context of galaxies. The rotation curve of a galaxy is the circular velocity plotted with the radius from the centre. From visible mass, galaxies should have a decreasing slope on the rotation curve, but as seen in observation they typically sport a rather flat rotation curve. This is interpreted as an indirect detection of dark matter.

**Satellite galaxy** Smaller galaxy orbiting a larger galaxy. See Sect. 2.2.

**Scale length** The length within which something has decreased with a factor \( e \). Used in describing for example the geometry of the Galactic disk(s) or the Galactic bar through density.

**Spectral energy distribution (SED)** The incoming flux of an object plotted with the wavelength or frequency.

**Supermassive black hole (SMBH)** Very massive (typically of mass \( \sim 10^6 \sim 10^9 \, M_\odot \)) black hole known to inhibit centres of galaxies and thought to derive from the early Universe.

**Tracer star** In this work, the term tracer star is used to represent the type of star which is used to probe the mass density in the Galaxy model by tracing their trajectories. Stars differ in type with changing mass and phase in evolution. The chosen tracer star in this work is M0III stars, which are bright red giants.

**Trailing spiral arms** When the spiral pattern is rotating in the sense that the arms seem to lag behind the motion. The opposite is called leading.