Epistemic Modalities as Fixed Points

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Abstract

Epistemic logics have been of considerable help in clarifying the reasoning needed for algorithms to maintain consistency in distributed databases, and for other communication protocols in multi-agent systems. But there are enough parameters – group size, synchronous versus asynchronous communication, and the reliability of communications – for basic problems to remain. The attainability of common knowledge or some approximation to it within a given environment is a typical concern.

A familar approach is to use a modal formula $C_{GK}\alpha$ to express the property that α is common knowledge to a group of agents G, and to give its intuitive meaning as the potentially unbounded conjunction $\bigwedge_{\{i\}} E^i_{GK}\alpha$, for $i = 1, 2, \ldots$, where $E_{GK}\alpha$ expresses the proposition $\forall j \in G, K_j\alpha$, and K_j is a modal operator for the knowledge of an agent j. $C_{GK}\alpha$ can also be defined as the greatest solution of the equivalence $C_{GK}\alpha \leftrightarrow E_{GK}\alpha \wedge E_{GK}C_{GK}\alpha$ under the usual ordering of entailment between propositions, and expressed as the propositional fixed point $\nu x.E_{GK}\alpha \wedge E_{GK}x$. However this focus on the formula $C_{GK}\alpha$ rather than the operator C_{GK} makes reasoning about protocols for realization of the operator at least indirect.

A little surprisingly, the usual modal epistemic operators, those for knowledge and belief, as well as the more sophisticated operators for common and distributed knowledge and belief can each be defined as a fixed point in a higher order modal logic. For example, in a logic with appropriate typing and semantics, the term $\nu K.\lambda\alpha.\alpha \wedge KK\alpha$ is an expression for the modal operator for knowledge itself. A challenge is dynamic systems which operationalize approximants to such epistemic operators. A short presentation will seek to promote discussion of this elementary ongoing work which began as a pragmatic review of some classical problems for asynchonous systems.