Continuous fragment of the mu-calculus

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Given a proposition letter p, we define the p-continuous fragment of the μ-calculus as the set of formulas of the μ-calculus which are Scott continuous with respect to p in the powerset algebra (with all other variables fixed). Equivalently, a formula φ is Scott continuous in p iff it is monotone and whenever the formula is true at a point in a model, we only need finitely many points where p is true in order to establish the truth of φ.

We provide a syntactic characterization of this fragment. More precisely, if P is a set of proposition letters, the set of formulas $SC(P)$ is defined by induction in the following way:

$$\varphi \ ::= \ \top \mid p \mid \psi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \Diamond\varphi \mid \mu x.\chi,$$

where p belongs to P, no proposition letters of ψ belong to P and χ belongs to $SC(P \cup \{x\})$. We show that the set $SC(\{p\})$ is precisely the p-continuous fragment of the μ-calculus. This proves a conjecture by Johan van Benthem. The technique is similar to the one used by M. Hollenberg (see, e.g., [1]) to show that a formula distributes over arbitrary unions iff it is equivalent to some $\langle\pi\rangle p$, where π is a p-free μ-program. The idea is to identify automata corresponding to this fragment and next to show that these automata give us the announced characterization.

References

[1] M. Hollenberg. Safety for Bisimulation in Monadic Second-Order Logic. *Logic Group Preprint Series 170*, Dept. of Philosophy, Utrecht University, 1996.