Invariant-Free Clausal Temporal Resolution

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One desirable property of a deductive system is that proof search should be efficiently mechanizable. The method of resolution, invented by J.A. Robinson in 1965, is an efficient refutation theorem proving method. Proof procedures for temporal logics have tended to be tableau or automata based rather than based on resolution principle. However, there are some interesting resolution methods for linear temporal logic that can be classified into two main groups. On the one hand, some methods (e.g. [3, 1]) are applied to clausal languages without positive eventualities (that is, formulas whose outermost operator is $\mathcal{U}$, in words, the until operator). On the other hand, some other methods (e.g. [2, 5, 6]) are applied to (clausal or non-clausal) languages that include positive eventualities. The former methods do not need any rule to resolve an eventuality, while the latter ones use a specialized *temporal resolution* rule that needs to look for an invariant formula. The invariant generation is the most costly part of this kind of approaches. In particular, in [6], the search of the invariant formula requires specialized graph search algorithms. [4].

In this paper, we introduce a new resolution method that is sound and complete for full PLTL (Propositional Linear Temporal Logic). Our method is based on the complete finitary sequent calculus for PLTL introduced in [7]. This calculus manages the eventualities in a way that avoids to search for an invariant formula. We first define a new notion of *temporal normal form* and prove that every PLTL-formula can be translated into a formula in this normal form, preserving satisfiability. Then we give a resolution method on clauses in temporal normal form. Our resolution mechanism simulates explicitly the transition from one world to the next one. Inside each world, only the classical resolution rule is applied. This is achieved by means of some rules that split the temporal formulas, in particular the eventualities, in a finite number of formulas. The obtained resolvents are directly in the required temporal normal form, hence further transformation of constituents is not needed. We believe that the main advantage of our proposal is that the temporal normal form and the resolution method are, respectively, very close to the classical CNF and the classical resolution method. Moreover, our invariant-free clausal resolution method finishes for any set of clauses deciding its (un)satisfiability, hence it gives raise to a new decision procedure for PLTL.

References

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