# Logic with temporally accessed iteration over finite structures

Alexei Lisitsa

Department of Computer Science, The University of Liverpool,

E-mail: {alexei}@csc.liv.ac.uk

We propose an extension of first-order logic with an iteration construct, which allows access to the iteration steps via first-order temporal logic. The syntax of FO+TAI (*first-order logic with temporally accessed iterations*) extends the standard syntax of first-order logic with the following construct. Let $\varphi(R, \bar{x})$ be a formula with free individual variables $\bar{x} = x_1, \ldots, x_k$ and free predicate variable $R$ of arity $k$. Let $\psi(\bar{z})$ be a first-order *temporal formula* with free individual variables $\bar{z} = z_1, \ldots, z_m$. Then

$$\tau := [\psi(\bar{z})][I_{R,\bar{x}}\varphi]\bar{t}$$

is also formula, where $\bar{t}$ is a tuple of terms of the same length as $\bar{z}$. As to the semantics, the formula expresses the fact that interpretation of $\bar{t}$ belongs to the relation defined by the temporal formula $\psi(\bar{z})$ over the sequence of interpretations obtained by iteration of the operator associated with $\varphi(R, \bar{x})$.

Resulting logic FO+TAI is interpreted over classical (non-temporal) finite structures and subsumes many logics with deterministic inductive operators, including logics with Least-Fixed Point, Inflationary Fixed-Point, Partial Fixed-Point operators [1] as well as with Partial Fixed-Point with general semantics [3] and ID-logic of non-monotone inductive definitions [2]. By explicit translation we show that FO+TAI is no more expressive than classical FO+PFP. At the same time FO+TAI provides with more natural and concise definitions for many properties of finite structures. Further, we demonstrate that adding the same temporal machinery to FO+LFP does not increase the expressive power of this logic either.

Finally, we disscuss further directions, including extensions of the proposed logic to the case of not necessarily finite structures (via temporal logics over ordinals) and to the case of non-deterministic inductive operators (via branching time temporal logics).

## References

[1] Anuj Dawar, Yuri Gurevich: Fixed point logics. Bulletin of Symbolic Logic 8(1): 65-88 (2002)

[2] Mark Denecker, Eugenia Ternovska: A Logic of Non-Monotone Inductive Definitions. ACM Transactions on Computational Logic, to appear Vol 9, N 2 (tentative)

[3] Stephan Kreutzer: Partial Fixed-Point Logic on Infinite Structures. CSL 2002: 337-351