An Analysis of Syntactic Fragments of Hybrid CTL with the ↓ Binder

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The branching-time temporal logic known as Computation Tree Logic, or CTL, was developed as a tool to reason about concurrent or nondeterministic systems. It is a very used logic for automated verification (model-checking) of computer software and hardware specifications, because of its linear-time model-checking complexity (lower than the model-checking complexity of the full μ-Calculus). CTL has the finite model property and its satisfiability problem is decidable, having EXPTIME-Complete complexity.

Hybrid logics are extensions of modal logics that allow explicit references to individual states of a model. Their goal is to extend the expressive power of ordinary modal logics, without losing their good properties, such as decidability. Nonetheless, the task of increasing the expressive power of a language without losing decidability is not so easy. For instance, one hybrid operator that is very expressive and very appealing is the ↓ operator. However, hybrid logics with this operator are generally undecidable. This has motivated research on decidable fragments of such logics.

In order to bring back decidability in the presence of the ↓ operator, the logic has to be somehow restricted. The restrictions fall into two broad categories: semantic restrictions and syntactic restrictions. Semantic restrictions impose conditions on the models in which the formulas of the logic are evaluated. These restrictions are usually to transitive linear models or to transitive tree models. Another semantic restriction involves limiting the out-degree of every state to a fixed upper-bound. Syntactic restrictions impose conditions on the construction of formulas, allowing only certain types of interaction between the operators of the language (mainly between ↓ and the other operators). An interesting syntactic restriction consists of the set of formulas that, when put in negation normal form, do not have a ↓ appearing inside the scope of any universal operator of the language.

Hybrid-CTL with the ↓ operator (HCTL(@,↓)) is complex not only from the perspective of decidability. Proving completeness of an axiomatization for this logic is also a difficult task. The general methodology to prove strong completeness of a given axiomatization for a certain modal logic is the construction of what are called *canonical models*. This methodology, however, can only prove strong completeness for compact logics. When the logic is non-compact, there are two main approaches. The first one is to prove weak completeness of an axiomatization through the use of *finitary canonical models*. The second one is to use infinitary rules (rules with a potentially infinite number of premises) in the axiomatization. The first approach, however, only works for logics with the finite model property. This leaves a gap in general methodologies for completeness proofs: there is no general method to prove completeness of a finitary axiomatization for a non-compact logic that lacks the finite model property. HCTL(@,↓) falls exactly in this category: it inherits non-compactness from CTL and the lack of the finite model property from the basic hybrid logic with the ↓ operator. It would be interesting then, similarly to what was done with the decidability issue, to find fragments of this logic such that weak completeness, for formulas in the particular fragment, of a finitary axiomatization could be proven using the general method of finitary canonical models.

Our goal in this presentation is to explore the hybrid extension of CTL with the ↓ operator with no restrictions on the shape of the models. In order to keep decidability, we analyze the syntactic fragment mentioned above (that was originally studied for the hybrid extension of the basic modal logic) in the presence of the CTL fixpoint operators. We call it the *reducible fragment*. We also present a second fragment, called *co-reducible fragment*, and a finitary axiomatization that can be proven weakly complete for the formulas in this fragment using a finitary canonical model. Finally, it would also be interesting to raise the debate on how to apply this kind of syntactic restriction in order to obtain a decidable fragment of the hybrid μ-Calculus with the ↓ binder.

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