Axiomatizations for modal fixpoint connectives

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Given a set Γ of modal formulas of the form $\gamma(x, \vec{p})$, where x occurs positively in γ, the language $\mathcal{L}_\sharp(\Gamma)$ is obtained by adding to the language of polymodal logic $\mathbf{K}$ connectives $\sharp_\gamma$, $\gamma \in \Gamma$. Each term $\sharp_\gamma$ is meant to be interpreted as the parametrized least fixed point of the functional interpretation of the term $\gamma(x)$. Examples of such languages are CTL and LTL. We consider the following problem: given Γ, construct an axiom system $\mathbf{K}_\sharp(\Gamma)$ which is sound and complete w.r.t. the concrete interpretation of the language $\mathcal{L}_\sharp(\Gamma)$ on Kripke frames. In the talk we give an effective solution for this problem. For every connective $\sharp_\gamma$ our procedure provides a bounded set of axioms and derivation rules. In many concrete cases we simply obtain the standard fixpoint axioms and rules.

If an algebraic perspective on modal logic is adopted, then $\mathbf{K}_\sharp(\Gamma)$ is a collection of Horn formulas which is finite whenever Γ is finite.

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