Axiomatizations for modal fixpoint connectives

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Given a set $\Gamma$ of modal formulas of the form $\gamma(x, \vec{p})$, where $x$ occurs positively in $\gamma$, the language $L_{\sharp}(\Gamma)$ is obtained by adding to the language of polymodal logic $K$ connectives $\sharp_{\gamma}, \gamma \in \Gamma$. Each term $\sharp_{\gamma}$ is meant to be interpreted as the parametrized least fixed point of the functional interpretation of the term $\gamma(x)$. Examples of such languages are CTL and LTL. We consider the following problem: given $\Gamma$, construct an axiom system $K_{\sharp}(\Gamma)$ which is sound and complete w.r.t. the concrete interpretation of the language $L_{\sharp}(\Gamma)$ on Kripke frames. In the talk we give an effective solution for this problem. For every connective $\sharp_{\gamma}$ our procedure provides a bounded set of axioms and derivation rules. In many concrete cases we simply obtain the standard fixpoint axioms and rules.

If an algebraic perspective on modal logic is adopted, then $K_{\sharp}(\Gamma)$ is a collection of Horn formulas which is finite whenever $\Gamma$ is finite.

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