Topics in Modal Logic 2022: Exam

20 December 2022

- Each question is worth 20 points
- Your grade will be based on your five best scores on the six exercises below.
- Generally (but not in Exercise 2) you may use any result from the lecture notes without proof, provided that you provide an explict reference, or a (concise) formulation of the result that you refer to.
- Good luck!

Exercise 1 Give an example of a pointed model $(\mathbb{S}, s)$ where *exactly one* of the following two formulas is true:

$$\begin{array}{rcl} \varphi & = & \nu x.\mu y.(p \wedge \Box x) \vee (\overline{p} \wedge \Box y) \\ \psi & = & \mu y.\nu x.(p \wedge \Box x) \vee (\overline{p} \wedge \Box y). \end{array}$$

Give a brief justification of your answer.

Exercise 2 Let S be some set, let $F : \wp(S) \to \wp(S)$ be a monotone operation, and let $\eta = \mu$ or $\eta = \nu$ (at your choice). Give a direct proof showing that the unfolding game $\mathcal{U}^\eta(F)$ enjoys positional determinacy.

Exercise 3 Give an explicit construction transforming an arbitrary nondeterministic parity stream automaton into an equivalent nondeterministic Büchi stream automaton. (You need to justify your answer but detailed proofs are not necessary.)

Exercise 4 Define a suitable notion of *equivalence* between parity formulas.

That is, given two parity formulas $\mathbb{G} = (V, E, L, \Omega, v_I)$ and $\mathbb{G}' = (V', E', L', \Omega', v'_I)$, write down natural conditions on a relation between V and V' such that, whenever there is a relation Z satisfying these conditions, $\mathbb{G}\langle v\rangle$ and $\mathbb{G}'\langle v'\rangle$ are equivalent for every pair $(v, v') \in Z$. (Recall that $\mathbb{G}\langle v\rangle$ is the parity formula $\mathbb{G}$, initialized at v, that is, $\mathbb{G}\langle v\rangle = (V, E, L, \Omega, v)$; and similarly for $\mathbb{G}'\langle v'\rangle$.) Supply a proof sketch for this equivalence. Try to make your definition as general as possible.

Exercise 5 Let $\mathbb{A} = (A, \Theta, \Omega, a_I)$ be a modal automaton relative to a set P of proposition letters, and let q be a proposition letter in P. As usual we write $C := \wp(\mathsf{P})$.

Assume that the state space A of $\mathbb{A}$ is partitioned as $A = A_0 \uplus A_1$ and that

- $a_I \in A_0$;

- $\Omega(a)$ is odd, for every $a \in A_0$;
- $\Theta(a,c) \in \mathtt{1ML}(A_1)$, for all $a \in A_1, c \in C$ (i.e., no $b \in A_0$ appears in any formula $\Theta(a,c)$ with $a \in A_1$);
- each occurrence of $b \in A_0$ in a formula $\Theta(a,c)$ is in the scope of a diamond (and hence, not in the scope of a box);
- $\Theta(a,c) = \bot$ whenever $a \in A_1$ and $q \in c$.

Let $\mathbb{S} = (S,R,V)$ be a tree model with root r, and assume that $\mathbb{A}$ accepts $(\mathbb{S},r)$. Then there is some *finite* subset $Q \subseteq V(q)$ such that $\mathbb{A}$ accepts $(\mathbb{S}[q \mapsto Q], r)$. (Hint: work towards an application of König's Lemma.)

Exercise 6 State the Janin-Walukiewicz Theorem, and provide a proof sketch (max 1 page). Make sure that you mention where in the proof properties of the one-step logics are used.