

Topics in Modal Logic (Fall 2025)

Tutorial Exercises 6

Exercise 1 (canonical one-step models)

Let $\alpha \in \text{ML}_\Lambda(A)$ be a one-step formula. Then α is one-step valid iff α holds at every A -canonical one-step model.

Exercise 2 (one-step soundness) Show that every substitution instance of a sound one-step rule is sound.

Exercise 3 (modalities for $T = Id \times Id$)

Consider the functor $T = Id \times Id$, with the modalities \bigcirc_0 and \bigcirc_1 . Interpreted in a T -model, the semantics of \bigcirc_i is given by

$$\mathbb{S}, s \Vdash \bigcirc_i \varphi \text{ if } \mathbb{S}, \pi_i \sigma(s) \Vdash \varphi,$$

where π_0 and π_1 are the projection functions from $S \times S$ to S .

Give a set of one-step rules for this modal signature which is one-step sound and complete.

Exercise 4 (one-step completeness) Let \mathbf{G} be a one-step complete one-step derivation system, and let Δ be a one-step valid rank-1 sequent. Show that Δ is derivable in \mathbf{G} .